



THE IMAGE FILTERING

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ABSTRACT

In image processing tasks, one of the quality improvement approaches is noise filtering. Moreover, the noise can be both in the form of additive and multiplicative interference. Most of the noise requires filtering based on optimal algorithms. This in turn requires computational costs. One of the effective recurrent filtering approaches is the Kalman filter. The paper considers the approach of constructing a block-shaped Kalman filter and methods of SRR (Super Resolution Reconstruction). image reconstruction.

INTRODUCTION

In image recognition tasks, one of the processing steps is noise filtering. Moreover, noises can be both in the form of additive and multiplicative interference [1-4]. Most of the noise requires filtering based on optimal algorithms. This in turn requires computational costs [5-7]. One of their effective approaches to filtering of the recurrent type is the Kalman filter, the advantage of which is the possibility of taking into account the covariance of forecast errors without imposing restrictions on the time interval in which the assimilation of previously obtained observational data (the assimilation window) occurs, as well as the possibility of a priori estimation of the accuracy of the results obtained by means of the algorithm itself [8-10].

first method: Kalman filtering

An image is fed to the input of the surveillance system [11]. The discrete system is described by the equations:

$$(k+1) = \Phi(k)x(k) + f + v(k), x(0) = x_0, \quad (1)$$

where $x(k) \in \mathbb{R}^n$ is a state vector; $\Phi(k)$ is an $n \times n$ matrix; f is an unknown constant vector; $v(k)$ is a white Gaussian random sequence with characteristics

$$M\{v(k)\} = 0, M\{v(k) v^T(j)\} Q(k) \delta_{(k,j)}. \quad (2)$$

The observation channel has the form $y(k) = H(k)x(k) + w(k)$,

$y(k) \in \mathbb{R}^l$ is a measurement vector; $H(k)$ is a matrix of dimension $l \times n$; $w(k)$ is a white Gaussian random sequence of measurement errors, with characteristics:



$M\{q(k)\}=0, M\{q(k) q^T (j)\}V(k) \delta_{(k,j)}$, for the matrices $(H(k), \Phi(k))$, the observability conditions are met. The vector x_0 is random and does not depend on the processes $w(k)$ and $v(k)$, while

$$M\{x(0)\}=(x_0)^T, M\{x(0)-(x_0)^T\}(x(0)-(x_0)^T)^T=P_0. \quad (4)$$

The discrete input information processing algorithm in the framework of Kalman filtering methods has the form [11]:

$$x_0 (| k-1) = \Phi(k) x_0 (k-1) + B(k)U(k) + D(k)F(k);$$

$$x_0 (k) = x_0 (| k-1) + \sum_{i=1}^N K_i (k) \{y_i (k) - H(k) x_0 (| k-1)\};$$

$$K_i (k) = S_i (k) P(k|k-1) H^T \{H(k) P(k|k-1) H^T (k) + V_{vi} [k]\}^{-1}; \quad (5)$$

$$P(| k-1) = G(k) V_w (k) G^T (k) + \Phi(k) P(k-1) \Phi^T (k);$$

$$P(k) = P(| k-1) - \sum_{i=1}^N K_i (k) H(k) P(| k-1), \quad i=1, \dots, N,$$

here $y_i (k)$ is the observation vector; $y_0 = H(n) x_0 (| n-1)$ is the vector of observation estimates; $x_0 (n)$ is the state vector estimate; $x_0 (k|k-1)$ - estimation of the state forecasting vector; $\Phi(k)$ is a transition matrix; $H(k)$ is the observation matrix; $K_i(k)$ - matrix of coefficients; $P(k|k-1)$ is the dispersion matrix of the state vector; $P(k)$ is the dispersion matrix for estimating the state vector; $U(k)$ is the control vector; $F(k)$ is the vector of measured signals from the output of the object; $B(k)$ is the matrix of control coefficients; $D(k)$ is the matrix of measurement coefficients; $S_i(k)$ - a sign of the type of meter or lack of measurements; $S_i(k) = 0$ [10,11].

The continuous Kalman filtering algorithm in time has the form [11]:

$$(dx_0)/dt = \Phi(t) x_0 (t) + B(t)U(t) + D(t)F(t) + \sum_{i=1}^N K_i (t) \{y_i (t) - H(t)x_0 (t)\};$$

$$(dP(t))/dt = V_w (t) + \Phi(t)P(t) + P(t) \Phi^T (t) - P(t) H^T (t) V_v (-1) (t) H(t)P(t), \quad (6)$$

where $y_i(t)$ is the observation vector; $y_0(t) = H(t)x_0(t)$ is the vector of estimates of observations; $x_0(t)$ is the state vector estimate; $\Phi(t)$ is a transition matrix; $P(t)$ is the correlation matrix; $H(t)$ is the observation matrix; $K_i (t) = S_i (t)P(t)H^T (t)V_v (-1) (t)$ - matrix of coefficients; $U(t)$ is the control vector; $F(t)$ is the vector of measured signals from the output of the object; $B(t)$ is the matrix of control coefficients; $D(t)$ is the matrix of measurement coefficients; indication of the type of meter or lack of measurements; $S_i(k) = 0$ [10].

The predicted value of the observed signal: $\hat{y}(k) = C(k)\Phi(k) x^*(k-1)$. The difference or discrepancy between the predicted and actually observed signals:

$$e(k) = y(k) - \hat{y}(k),$$

(7)

when $P(k-1) C^T (k) \times (C(k)P(k-1) C^T (k) + Q_M (k))^{-1} = K(k)$ is Kalman gain factor [11].

Extrapolation of the state vector of the system according to the state vector estimate and applied to the control vector from step $(k-1)$ to step k : $x^*(k) = \Phi(k) x^*(k-1) + K(k)e(k)$ is the posterior estimate of the state vector for the k th frame, the dimension of the vector is determined by the order of the filter.

Updating the estimation of the correlation matrix of filtering errors has the form [11]:

$$P(k) = \Phi(k) [P(k-1) - K(k)C(k)P(k-1)] \Phi^T (k) + Q_M (k)$$

$Q_M (k)$ is covariance matrix of some random variable; therefore, its trace is non-negative. The trace minimum is reached when the last term is zeroed:

$$K(k) = P(k-1)H^T (k)S^(-1) (k)$$



This matrix is the desired one and, when used as a matrix of coefficients in the Kalman filter, it minimizes the sum of the mean square errors of the state vector estimate.

It is assumed that there are no deterministic changes in the coefficients; therefore, the transition matrix Φ is unit: $\Phi(k) = I$. The observation matrix is the vector of the content of the filter delay line $u(k)$. Thus, the filter output signal is the predicted value of the observed signal, and the exemplary adaptive filter signal $d(k)$ acts as the observed signal itself. In this case, the observation noise is an error in reproducing an exemplary signal, and the QM matrix turns into a scalar parameter the mean square error signal [11].

If a stationary random process is filtered, the coefficients of the optimal filter are constant and $Q_p = 0$ can be taken. To enable the filter to track slow changes in the statistics of the input signal, a diagonal matrix can be used as the covariance matrix. As a result, the above formulas (2-5) take the following form:

$y(k) = u^T(k) w^*(k-1)$ is a posterior filter output signal (predicted value of the reference signal);

$e(k) = d(k) - y(k)$ – filter residual;

$K(k) = (P(k-1)u(k)) / (u^T(k)P(k-1)u(k) + Q_M)$ is a Kalman gain factor;

$w^*(k) = w^*(k-1) + K(k)e(k)$ are estimates of filter coefficients;

$P(k) = P(k-1) - K(k) u^T P(k-1) + Q_p$ are estimation errors.

The initial value of the vector w is usually assumed to be zero, and a diagonal matrix of the form C [11] is used as the initial estimate of the matrix P . The initial value of the vector w is usually taken to be zero, and the

diagonal matrix is used as the initial estimate of the matrix P .

When processing images with the Kalman filter, there was a problem associated with the large size of the system matrices, which, accordingly, led to an increase in the time and volume of calculations, as well as the consumption of computing resources. One of the promising approaches to solving the problem of recognition under conditions of anomalous observations, shading of objects and the presence of affected areas is the use of super resolution reconstruction methods SRR (Super Resolution Reconstruction), block image processing.

SECOND METHOD: SUPER RESOLUTION RECONSTRUCTION METHOD

The SRR algorithm transforms several low-resolution corrupted images y_k , where $k=1, \dots, K$ at the input of the block filter, by comparing with a low-resolution reference image with fractional pixel precision. Image size $M \times M$ pixels. The evaluation of low resolution images is carried out by means of a high resolution image x_k [12].

$$x_{k+1} = F_k x_k + w_k, \quad (8)$$

where x_{k+1} is a high-resolution image obtained from the previous image x_k by moving the camera and/or object in the process of obtaining images; F_k is an $N_2 \times N_2$ shift matrix that characterizes image shifts; w_k is Gaussian noise with zero mathematical expectation and covariance matrix $Q_k = \sigma_k^2 (Q) I$ with size $N_2 \times N_2$.

$$| \blacksquare (Y_1 @ Y_2 @ (Y_k \dots)) |$$

$$| \blacksquare (H_1 @ H_2 @ (H_k \dots)) | x_k + | \blacksquare ([H = E]_1 @ E_2 @ (E_k \dots)) |$$

$$y_k = H_k x_k + E_k, \quad (9)$$

where y_k is the observed low-resolution image; x_k is the high resolution image from



which y_k is derived; H_k is the $M_2 \times N_2$ decimation matrix that transforms x_k into y_k ; E_k – additive zero mean Gaussian noise with zero mathematical expectation and positive definite autocorrelation matrix $R_k = \sigma_k^2 (R)^2$ I , size $M_2 \times M_2$. Moreover, the matrices H_k , F_k , R_k and Q_k , which define the state space system, are considered to be known [12].

The autocorrelation matrix may be chosen as the identity matrix if a priori data on additive noise is not given. This choice is consistent with the assumption that the noise is white, which is typically the case for many reconstruction problems, including super-resolution applications.

$$E\{EE^T\} = [■(R_1@0) \cdots ■(0@R_k)]^T (10)$$

where we have determined the autocorrelation of a Gaussian random vector.

Having (8) and (9) in the classical form, one can solve the filtering problem, since the additive noise is equal to a Gaussian random process with a zero autocorrelation matrix, performing several algebraic transformations, we obtain that the maximum likelihood estimate reduces to a weighted least squares estimate of the form

$$\hat{x}_k = (\text{argmax})_T x^T [x_k] (y/x) = [\text{argmax}_T x] [y_k - Hx]^T R [y_k - Hx] \quad (11)$$

$$\theta \hat{x}_k = x_k, \quad (12)$$

where

$$\theta = H^T R H = \sum_{k=1}^N [H_k^T R_k H_k] \quad (13)$$

$$x_k = H^T R y = \sum_{k=1}^N [H_k^T R_k y_k] \quad (13)$$

Thus, we get a high resolution image.

Applying the block form of the Kalman filter, one can solve the problem of image reconstruction in the presence of object shading, the occurrence of affected areas of images and anomalous observations, reducing low-resolution images (9) to the form (10), subject to additivity of noise.

DISCUSSION

When using the Kalman filter to solve adaptive filtering problem, the monitored process is the vector of coefficients optimal filter w . It is assumed that there are no deterministic changes in the coefficients; therefore, the transition matrix Φ is the identity matrix: $\Phi(k) = I$. The observation matrix is the vector of the content filter delay line $u(k)$. Thus, the filter output is predicted value of the observed signal, and model signal of the adaptive filter $d(k)$ acts as the observed signal itself. In this case, the observation noise is an error in reproducing an exemplary signal, and the matrix Q_M turns into a scalar parameter the mean square of the error signal.

When processing images with a Kalman filter of a block form, 3x3 and 4x4 blocks were used. The Q_M error covariance matrix is constantly updated. The results of image processing and the standard deviation of the error were obtained. The gain of the Kalman filter is obtained based on the covariance matrix, which minimizes the sum of mean square errors of the state vector estimate. In this case, the filter output signal represents predicted value of observed signal, and the model signal of adaptive filter acts as the observed signal itself.

CONCLUSION

Using the Kalman filter, objects necessary for processing in the recognition process



were selected, some of the objects were deleted. This is one of the disadvantages of the Kalman filter associated with the high sensitivity of the resulting estimate with respect to the influence of abnormal effects such as abnormal observations, shadowing of objects, and the occurrence of affected

areas of images. When processing images with the Kalman filter, a problem arose due to the large size of the system matrices, which accordingly led to an increase in the time and volume of calculations, as well as the consumption of computing resources.

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