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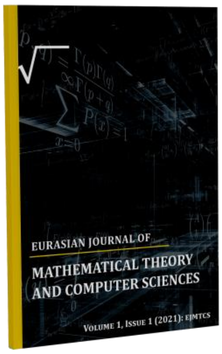

MATEMATIK INDUKSIYA METODI YORDAMIDA TENGSIZLIKLARNI ISBOTLASH

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ABSTRACT

Usbu maqolada natural parametrga bog'liq ayrim tengsizliklarni matematik induksiya metodi yordamida isbotlash o'rganilgan.

Matematika fanining eng qiziqarli mavzularidan bir tengsizliklar va uni isbotlash hisoblanadi. Tengsizliklarni isbotlashni o'quvchilarga o'rgatishda isbotlashning bir necha usullarini ko'rsatib berish fan o'qituvchisining asosiy vazifasi hisoblanadi. Bu esa o'quvchida bir muammoni hal etishda turli yo'nalishlarda fikrlash qobilyatini shakllantiradi. Ushbu maqolada tengsizliklarni isbotlashning matematik induksiya metodi (keyingi o'rinlarda MIM deb yoziladi) orqali isbotlash bayon etilgan.

Bizga ma'lumki [1, 2], MIMining mohiyati quyidagicha:

Agar tasdiqlash ketma-ketligi mavjud bo'lsa, birinchi tasdiq to'g'ri va har bir to'g'ri tasdidan so'ng to'g'ri tasdiq mavjud bo'lsa, ketma-ketlikdagi barcha tasdiq to'g'ri hisoblanadi.

Shunday qilib, MIM yordamida isbotlash ikkita teoremadan iborat.

1-teorema . $n = 1$ uchun tasdiq to'g'ri.

2-teorema. Ixtiyoriy $n=k$ uchun tasdiq to'g'ri deb faraz qilinsa, u holda, navbatdagi $n=k+1$ natural son uchun tasdiq to'g'ri deb hisoblanadi.

Agar ikkala ushbu teoremalar isbotlangan bo'lsa, matematik induksiya tamoyili (keying o'rinlarda MIT deb keladi)ga asoslangan holda, tasdiq ixtiyoriy n natural son uchun to'g'ri deb xulosa qilinadi.

Eslatma. Barcha natural sonlar uchun emas, balki n dan katta yoki teng m natural sonlar uchun induksiya bo'yicha tasdiqni isbotlash zarur bo'ladi. Bunday holda isbotlash quyidagicha bajariladi.

1-teorema. $n = m$ da tasdiq to'g'ri.

2-teorema. $n=k$ da tasdiq to'g'ri berilgan, $k \geq m$. $n = k+1$ da tasdiq o'rinli ekanligini isbotlash lozim.

1-masala. Faraz qilaylik, a, b – katetlar uzunligi, c – to'g'ri burchak gipotenuzasining uzunligi. U holda barcha $n \geq 2$ natural sonlar uchun

$$a^n + b^n \leq c^n \quad (1)$$

o'rinlidir.

1-teorema. Pifagor teoremasidan $a^2 + b^2 = c^2$ tenglik hosil qilinadi.

1-teorema isbotlandi.

2-teorema. (1) tengsizlik $n = k$, $k \geq 2$, da bajariladi deb faraz qilaylik,ya'ni $a^k + b^k \leq c^k$. to'gri. U holda (4.1) tengsizlik $n = k + 1$: $a^{k+1} + b^{k+1} \leq c^{k+1}$ bajariladi.

Haqiqatdan:

$$a^{k+1} + b^{k+1} = a^k \cdot a + b^k \cdot b < a^k \cdot c + b^k \cdot c = c \cdot (a^k + b^k) \leq c \cdot c^k = c^{k+1}.$$

2-teorema isbotlandi.

1-teorema va 2-teoremalardan ixtiyoriy $n \geq 2$ natural son uchun (1) tengsizlikning o'rinli ekanligi kelib chiqadi.

2-masala. Agar $a \geq -1$ bo'lsa, u holda

$$(1+a)^n \geq 1 + na, \forall n \in N \qquad (2)$$

ekanligini isbotlang.

1-teorema. $n = 1$ da quidagiga ega bo'lamiz:

(2) tengsizlikning chap qismi: $(1+a)^1 = (1+a)$

(2) tengsizlikning o'ng qismi: $(1 + 1 \cdot a) = 1 + a$ ga teng. 1-teorema isbotlandi.

2-teorema. Faraz qilaylik, (2) tengsizlik $n = k$: $(1+a)^k \geq 1 + ka$ bajarilsin. U holda tengsizlik

$n = k + 1$ uchun o'rinli bo'ladi: $(1+a)^{k+1} \geq 1 + (k+1)a$.

Haqiqatdan, $(1+a) \geq 0$ bo'lganda

$$(1+a)^{k+1} = (1+a)^k (1+a) \geq (1+ka)(1+a) = 1 + (k+1)a + \underbrace{ka^2}_{\geq 0} \geq 1 + (k+1)a$$

bajariladi.

2-teorema isbotlandi. 1- va 2- teoremalardan ixtiyoriy natural son uchun (2) tengsizlik bajariladi.

1-eslatma. Ma'lumki, $\left\{\left(1+\frac{1}{n}\right)^n\right\}_{n=1}^{\infty}$ ketma-ketlik chegaraga ega bo'lib, ***e*** soni deyiladi:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e = 2{,}718...$$

Ketma-ketlik xossasidan:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\cdot\underbrace{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)}_{=1}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}=e$$

. (2.1) ekanligi ma'lum bo'ladi.

$x_n=\left(1+\frac{1}{n}\right)^n \quad (n=1,2,\ldots)$ ketma-ketlikning monoton o'sishini, $y_n=\left(1+\frac{1}{n}\right)^{n+1} \quad (n=1,2,\ldots)$ ketma-ketlikning esa monoton kamayishini isbotlang.

Isbotlash. Agar $x_n<x_{n+1}$ bo'lsa, bu $\frac{x_{n+1}}{x_n}>1$ tengsizlikka teng kuchli, shuning uchun $\{x_n\}$ ketma-ketlik monoton o'sadi. Quyidagi tengsizlikni isbotlash lozim:

$$\frac{x_{n+1}}{x_n}=\frac{\left(1+\frac{1}{n+1}\right)^{n+1}}{\left(1+\frac{1}{n}\right)^{n}}>1 \quad (n=1,2,\ldots)$$

. (2.2)

Agar $y_{n-1}>y_n$ bo'lsa, bu $\frac{y_n}{y_{n-1}}<1$ tengsizlikka teng kuchli, u holda $\{y_n\}$ ketma-ketlik monoton kamayadi. Quyidagi tengsizlikni isbotlang:

$$\frac{y_n}{y_{n-1}}=\frac{\left(1+\frac{1}{n}\right)^{n+1}}{\left(1+\frac{1}{n-1}\right)^{n}}<1 \quad (n=1,2,\ldots)$$

. (2.3)

(2.2) tengsizlikni isbotlaymiz:

$$\frac{x_{n+1}}{x_n}=\frac{\left(1+\frac{1}{n+1}\right)^{n+1}}{\left(1+\frac{1}{n}\right)^{n}}=\frac{(n+2)^{n+1}n^n}{(n+1)^n(n+1)^{n+1}}\cdot\frac{n\cdot(n+1)}{n\cdot(n+1)}=\frac{((n+2)n)^{n+1}}{(n+1)^{2(n+1)}}\cdot\frac{(n+1)}{n}=$$

$$=\left(\frac{n^2+2n+1-1}{(n+1)^2}\right)^{n+1}\cdot\frac{(n+1)}{n}=\underbrace{\left(1-\frac{1}{(n+1)^2}\right)^{n+1}}\cdot\frac{(n+1)}{n}>$$

$$>\left(1-\frac{1}{(n+1)}\right)\cdot\frac{(n+1)}{n}=1 \quad \Rightarrow \quad \frac{x_{n+1}}{x_n}>1.$$

(2.3) tengsizlik (2.2) tengsizlikka o'xshash isbotlanadi:

$$\frac{y_n}{y_{n-1}}=\frac{\left(1+\frac{1}{n}\right)^{n+1}}{\left(1+\frac{1}{n-1}\right)^{n}}=\frac{(n+1)^n\cdot(n-1)^n}{(n^2-1+1)^n}\cdot\frac{n+1}{n}=\frac{1}{\underbrace{\left(1+\frac{1}{n^2-1}\right)^n}_{(4.2)}}\cdot\frac{n+1}{n}<$$

$$<\frac{1}{1+\frac{n}{n^2-1}}\cdot\frac{n+1}{n}=\frac{n^3+n^2-n-1}{n^3+n^2-n}<1 \quad \Rightarrow \quad \frac{y_n}{y_{n-1}}<1.$$

$x_n=\left(1+\frac{1}{n}\right)^n \quad (n=1,2,\ldots)$ ketma-ketlik qa'tiy monoton o'sadi $y_n=\left(1+\frac{1}{n}\right)^{n+1} \quad (n=1,2,\ldots)$ ketma-ketlik qa'tiy monoton kamayadi, (2.1) tengsizlikni hisobga olgan holda quyidagi tengsizlikni hosil qilamiz:

$$\left(1+\frac{1}{n}\right)^n<e<\left(1+\frac{1}{n}\right)^{n+1}. \qquad (2.4)$$

2-eslatma. (2.2) tengsizlikni MIM bilan isbotlaymiz. Bu tengsizlik quyidagi tengsizlikka teng kuchli:

$$\left(1+\frac{1}{n}\right)^n<\left(1+\frac{1}{n+1}\right)^{n+1}. \qquad (2.2.1)$$

1-teorema. $n=1$ da $\left(1+\frac{1}{1}\right)^1=2<2{,}25=\left(1+\frac{1}{1+1}\right)^{1+1}$ ega bo'lamiz

2-teorema. (2.2.1) tengsizlikning *n=k* da bajarilishi berilgan:

$\left(1+\frac{1}{k}\right)^{k} < \left(1+\frac{1}{k+1}\right)^{k+1}$. (2.2.1) tengsizlikning $n=k+1$ da bajarilishini isbotlash lozim:

$$\left(1+\frac{1}{k+1}\right)^{k+1} < \left(1+\frac{1}{k+2}\right)^{k+2}.$$

Isbotlash.

$$\left(1+\frac{1}{k+1}\right)^{k+1} = \left(1+\frac{1}{k+1}\right)^{k+1}\frac{\left(1+\frac{1}{k+2}\right)^{k+2}}{\left(1+\frac{1}{k+2}\right)^{k+2}} =$$

$$=\left(1+\frac{1}{k+2}\right)^{k+2}\frac{(k+2)^{k+1}(k+2)^{k+2}}{(k+1)^{k+1}(k+3)^{k+2}} = \left(1+\frac{1}{k+2}\right)^{k+2}\frac{((k+2)^2)^{k+1}(k+2)}{((k+1)(k+3))^{k+1}(k+3)} =$$

$$=\left(1+\frac{1}{k+2}\right)^{k+2}\left(\frac{k^2+4k+4}{k^2+4k+3}\right)^{k+1}\cdot\frac{k+2}{k+3} =$$

$$\left(1+\frac{1}{k+2}\right)^{k+2}\cdot\frac{1-\frac{1}{k+3}}{\left(1-\frac{1}{(k+2)^2}\right)^{k+1}} <$$

$$\underbrace{\left(1-\frac{1}{(k+2)^2}\right)^{k+1} > 1-\frac{k+1}{(k+2)^2}}_{(4.2)}$$

$$<\left(1+\frac{1}{k+2}\right)^{k+2}\frac{\left(1-\frac{1}{k+3}\right)}{1-\frac{k+1}{(k+2)^2}} = \left(1+\frac{1}{k+2}\right)^{k+2}\frac{(k+2)^3}{(k+3)(k^2+3k+3)} =$$

$$=\left(1+\frac{1}{k+2}\right)^{k+2}\underbrace{\frac{k^3+6k^2+12k+8}{k^3+6k^2+12k+9}}_{<1} < \left(1+\frac{1}{k+2}\right)^{k+2}.$$

2-teorema isbotlandi.

MITga ko'ra ixtiyoriy n natural son uchun (2.2.1) tengsizlik bajariladi.

3-masala. Tengsizlikni isbotlang

$$(1+x_1)\cdot(1+x_2)\cdot\ldots\cdot(1+x_n) \geq 1+x_1+x_2+\ldots+x_n,\ \forall n\in N, \qquad (3)$$

Bu yerda $x_i, i=1,\ldots,n$, – (-1) dan katta bo'lgan bir xil ishorali son.

1-teorema. $n=1$ da $1+x_1=1+x_1$ ga ega bo'lamiz. 1-teorema isbotlandi.

2-teorema. (3) tengsizlikning *n=k* da bajarilishi berilgan:

$$(1+x_1)\cdot(1+x_2)\cdot\ldots\cdot(1+x_k)\ge 1+x_1+x_2+\ldots+x_k.$$

(3) tengsizlikning *n = k + 1* da bajarilishini isbotlash lozim:

$$(1+x_1)\cdot(1+x_2)\cdot\ldots\cdot(1+x_k)\cdot(1+x_{k+1})\ge 1+x_1+x_2+\ldots+x_k+x_{k+1}.$$

Isbotlash.

$$\underbrace{(1+x_1)\cdot(1+x_2)\cdot\ldots\cdot(1+x_k)}_{\ge\ 1+x_1+x_2+\ldots+x_k}\cdot(1+x_{k+1})\ge(1+x_1+x_2+\ldots+x_k)\cdot(1+x_{k+1})=$$

$$=1+x_1+\ldots+x_k+x_{k+1}+\underbrace{x_1x_{k+1}+\ldots+x_kx_{k+1}}_{\ge\ 0}\ge 1+x_1+\ldots+x_k+x_{k+1}.$$

2-teorema isbotlandi. MITga ko'ra ixtiyoriy *n* natural son uchun (3) tengsizlik bajariladi.

4-masala. Tengsizlikni isbotlang:

$$\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2n-1}{2n}<\frac{1}{\sqrt{2n+1}},\quad \forall n\in N. \qquad (4)$$

1-teorema. $n = 1$ da: $\frac{1}{2}=\frac{1}{\sqrt{2^2}}=\frac{1}{\sqrt{4}}<\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{2\cdot1+1}}$. 1-teorema isbotlandi.

2-teorema. *n = k* da berilgan: $\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2k-1}{2k}<\frac{1}{\sqrt{2k+1}}.$

$$\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2k-1}{2k}\cdot\frac{2(k+1)-1}{2(k+1)}<\frac{1}{\sqrt{2(k+1)+1}}=\frac{1}{\sqrt{2k+3}}$$ tengsizlikni isbotlash lozim.

Isbotlash.

$$\underbrace{\frac{1}{2}\cdot\frac{3}{4}\cdot\ldots\cdot\frac{2k-1}{2k}}_{<\frac{1}{\sqrt{2k+1}}}\cdot\frac{2(k+1)-1}{2(k+1)}<\frac{1}{\sqrt{2k+1}}\cdot\frac{2k+1}{2(k+1)}\cdot\frac{\sqrt{2k+3}}{\sqrt{2k+3}}=$$

$$=\frac{1}{\sqrt{2k+3}}\cdot\frac{\sqrt{2k+1}\cdot\sqrt{2k+3}}{\sqrt{(2k+2)^2}}=\frac{1}{\sqrt{2k+3}}\cdot\underbrace{\sqrt{\frac{4k^2+8k+3}{4k^2+8k+4}}}_{<\ 1}<\frac{1}{\sqrt{2k+3}}.$$

2-teorema isbotlandi. MITga ko'ra ixtiyoriy *n* natural son uchun (4) tengsizlik bajariladi.

5-masala. Tengsizliklarni isbotlang:

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}>\sqrt{n},\quad \forall n\in N: n\geq 2 \quad ; \qquad (5.1)$$

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}<2\sqrt{n},\quad \forall n\in N: n\geq 2 \quad . \qquad (5.2)$$

(5.1) tengsizlikni isbotlaymiz.

1-teorema. *n = 2 da*: $1+\frac{1}{\sqrt{2}}=\frac{\sqrt{2}+1}{\sqrt{2}}>\frac{1+1}{\sqrt{2}}=\sqrt{2}$ ega bo'lamiz. 1-teorema isbotlandi.

2-teorema. *n = k d* $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}>\sqrt{k}$ tengsizlik berilgan.

$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$ tengsizlikni isbotlash lozim.

Isbotlash. $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>\sqrt{k}+\frac{1}{\sqrt{k+1}}=$

$=\frac{\sqrt{k^2+k}+1}{\sqrt{k+1}}>\frac{\sqrt{k^2}+1}{\sqrt{k+1}}=\sqrt{k+1}.$ 2-teorema isbotlandi. MITga ko'ra ixtiyoriy $n\geq 2$ natural son uchun (4.5) tengsizlik bajariladi.

Endi (5.2) tengsizlikni isbotlaymiz.

2-teorema. *n = 2* da: $1+\frac{1}{\sqrt{2}}=\frac{\sqrt{2}+1}{\sqrt{2}}<\frac{2+2}{\sqrt{2}}=2\sqrt{2}$. 1-teorema isbotlandi.

2-teorema. (5.2) tengsizlik *n = k* da bajarilishi berilgan:

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}<2\sqrt{k},\ k\geq 2.$$

n = k + 1 da : $1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}<2\sqrt{k+1}$ bajariladi.

Isbotlash. $\underbrace{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{k}}}_{<2\sqrt{k}}+\frac{1}{\sqrt{k+1}}<$

$$<\left(2\sqrt{k}+\frac{1}{\sqrt{k+1}}\right)\cdot\frac{\sqrt{k+1}}{\sqrt{k+1}}=\sqrt{k+1}\cdot\left(\frac{2\sqrt{k^2+k}+1}{k+1}\right)\cdot\quad \frac{2\sqrt{k^2+k}+1}{k+1}<2$$

tengsizlikning $\sqrt{4k^2+4k}+1<2k+2 \Leftrightarrow \sqrt{4k^2+4k}<2k+1$ tengsizlikka teng kuchli ekanligini isbotlash lozim. Oxirgi tengsizlik quyidagi tengsizlik va tenglikdan kelib chiqadi: $\sqrt{4k^2+4k}<\sqrt{4k^2+4k+1}=2k+1,\ \forall k\in N$.

2-teorema isbotlandi. MITga ko'ra ixtiyoriy $n\geq 2$ natural son uchun (5.2) tengsizlik bajariladi.

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\ldots+\frac{1}{\sqrt{n}}>\underbrace{\frac{1}{\sqrt{n}}+\frac{1}{\sqrt{n}}+\ldots+\frac{1}{\sqrt{n}}}_{n}=\frac{n}{\sqrt{n}}=\sqrt{n}$$

Eslatma: .

6-masala. Tengsizlikni isbotlang

$$n^{n+1}>(n+1)^n,\quad \forall n\in N: n\geq 3. \qquad (6)$$

1-teorema. *n = 3 da*: $3^4=81>64=4^3$. 1-teorema isbotlandi.

2-teorema. *n* = *k* da $k^{k+1}>(k+1)^k$ tengsizlikning bajarilishi berilgan.

$(k+1)^{k+2}>(k+2)^{k+1}$ tengsizlikning bajarilishini isbotlash lozim.

Isbotlash. Agar $k^{k+1}>(k+1)^k$ bo'lsa, u holda $1>\frac{(k+1)^k}{k^{k+1}}$ tengsizlik bajariladi. Oxirgi tengsizlikni musbat $(k+1)^{k+2}$ songa ko'paytirib, quyidagi tenglik va tengsizlik zanjirini hosil qilamiz:

$$(k+1)^{k+2}>\frac{(k+1)^k\cdot(k+1)^{k+2}}{k^{k+1}}=\frac{((k+1)^2)^{k+1}}{k^{k+1}}=\left(\frac{k^2+2k+1}{k}\right)^{k+1}=$$

$$=\left(k+2+\frac{1}{k}\right)^{k+1}>\left|\ \frac{1}{k}>0\ \right|>(k+2)^{k+1}$$. 2-teorema isbotlandi.

MITga ko'ra ixtiyoriy $n\geq 3$ natural son uchun (6) tengsizlik bajariladi.

7-masala. Tengsizliklarni isbotlang:

$$x_n=\underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{n+1\ \ ildiz}<\sqrt{2}+1,\quad \forall n\in N \qquad ; \qquad (7.1)$$

$$x_n=\underbrace{\sqrt{4+\sqrt{4+\ldots.+\sqrt{4}}}}_{n\ \ ildiz}<3,\quad \forall n\in N \qquad . \qquad (7.2)$$

(7.1) tengsizlikni isbotlaymiz.

1-teorema. *n = 1* da

$x_1 = \sqrt{2+\sqrt{2}} < \sqrt{2+2\sqrt{2}+1} = \sqrt{(\sqrt{2}+1)^2} = \sqrt{2}+1$ ega bo'lamiz. 1-teorema isbotlandi.

2-teorema. *n = k* uchun (7.1) tengsizlik berilgan :

$$x_k = \underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{k+1} < \sqrt{2}+1 .$$

*n = k+*1 tengsizlikni isbotlash lozim:

$$x_{k+1} = \underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{k+2 \quad ildiz} < \sqrt{2}+1 .$$

Isbotlash.

$$x_{k+1} = \underbrace{\sqrt{2+\sqrt{2+\ldots+\sqrt{2}}}}_{k+2 \quad ildiz \ <\sqrt{2}+1} < \sqrt{2+\sqrt{2}+1} < \sqrt{2+2\sqrt{2}+1} = \sqrt{2}+1 .$$

2-teorema isbotlandi. MITga ko'ra ixtiyoriy ***n*** natural son uchun (7.1) tengsizlik bajariladi.

Endi (7.2) tengsizlikni isbotlaymiz.

1-teorema. *n = 1 da* $x_1 = \sqrt{4} < \sqrt{9} = 3$ ega bo'lamiz. 1-teorema isbotlandi.

2-teorema. *n = k* da (7.1) tengsizlikning o'rinli ekanligini berilgan:

$$x_k = \underbrace{\sqrt{4+\sqrt{4+\ldots.+\sqrt{4}}}}_{k \quad ildiz} < 3 .$$

*n = k+*1 da tengsizlikni isbotlash lozim:

$$x_{k+1} = \underbrace{\sqrt{4+\sqrt{4+\ldots.+\sqrt{4}}}}_{k+1 \quad ildiz} < 3 .$$

Isbotlash.

$$x_{k+1} = \underbrace{\sqrt{4+\sqrt{4+\ldots.+\sqrt{4}}}}_{k+1 \quad ildiz} < \sqrt{4+3} = \sqrt{7} < 3$$

2-teorema isbotlandi. MITga ko'ra (7.2) tengsizlik ixtiyoriy ***n*** natural son uchun bajariladi.

8-masala. Ixtiyoriy ***n*** natural son uchun

$$|\sin nx| \le n|\sin x| \qquad (8)$$

tengsizlikni isbotlang

1-teorema. *n* = 1 da : $|\sin 1x| = 1\cdot|\sin x|$ 1-teorema isbotlandi.

2-teorema. *n = k* da $|\sin kx| \le k|\sin x|$ tengsizlikning bajarilishi berilgan.

$\left|\sin(k+1)x\right| \le (k+1)\left|\sin x\right|$ tengsizlikning bajarilishini isbotlash lozim.

Isbotlash. $\left|\sin(k+1)x\right| = \left|\sin kx\cos x + \sin x\cos kx\right| \le$

$$\le \underbrace{\left|\sin kx\right|}_{\le k|\sin x|}\cdot\underbrace{\left|\cos x\right|}_{\le 1} + \left|\sin x\right|\cdot\underbrace{\left|\cos kx\right|}_{\le 1} \le (k+1)\left|\sin x\right|.$$

2-teorema isbotlandi.

MITga ko'ra ixtiyoriy ***n*** natural son uchun (8) tengsizlik bajariladi.

9-masala. Ixtiyoriy ***n*** natural sonda

$$\frac{1}{n+1}+\frac{1}{n+2}+\ldots+\frac{1}{3n+1}>1 \qquad (9)$$

tengsizlikni isbotlash lozim.

$S_k=\dfrac{1}{k+1}+\dfrac{1}{k+2}+\ldots+\dfrac{1}{3k+1}$ orqali belgilaymiz.

1-teorema. $n = 1$ da: $S_1=\dfrac{1}{1+1}+\dfrac{1}{1+2}+\dfrac{1}{3\cdot 1+1}=\dfrac{13}{12}>1$ ga ega bo'lamiz. 1-teorema isbotlandi.

2-teorema. $n = k$ da quyidagi tengsizlikning bajarilishi berilgan:

$$S_k=\frac{1}{k+1}+\frac{1}{k+2}+\ldots+\frac{1}{3k+1}>1.$$

Quidagi tengsizlikning bajarilishini isbotlang

$$S_{k+1}=\frac{1}{k+2}+\frac{1}{k+3}+\ldots+\frac{1}{3k+1}+\frac{1}{3k+2}+\frac{1}{3k+3}+\frac{1}{3k+4}>1.$$

Isbotlash.

$$S_{k+1}=\frac{1}{k+2}+\frac{1}{k+3}+\ldots+\frac{1}{3k+1}+\frac{1}{3k+2}+\frac{1}{3k+3}+\frac{1}{3k+4}+\left(\frac{1}{k+1}-\frac{1}{k+1}\right)=$$

$$=\underbrace{\frac{1}{k+1}+\frac{1}{k+2}+\frac{1}{k+3}+\ldots+\frac{1}{3k+1}}_{=S_k>1}+\underbrace{\frac{1}{3k+2}+\frac{1}{3k+3}+\frac{1}{3k+4}-\frac{1}{k+1}}_{>0}>1.$$

" >0 " tengsizlik quyidagicha kelib chiqadi:

$$\frac{1}{3k+2}+\frac{1}{3k+3}+\frac{1}{3k+4}-\frac{1}{k+1}=\frac{1}{3k+2}+\frac{1}{3k+4}-\frac{2}{3k+3}=$$

$$=\frac{(3k+4)(3k+3)+(3k+2)(3k+3)-(6k+4)(3k+4)}{(3k+2)(3k+3)(3k+4)}=$$

$$= \frac{2}{(3k+2)(3k+3)(3k+4)} > 0$$

. 2-teorema isbotlandi.

MITgga ko'ra ixtiyoriy ***n*** natural son uchun (9) tengsizlik bajariladi.

Mustaqil yechish uchun masalalar.

1. $3^n - 2^n \geq n, \forall n \in N$.
2. $\left(\frac{n^2+1}{n^2}\right)^n \geq \frac{n+1}{n}, \forall n \in N$.
3. $\left(1+\frac{1}{n}\right)^{n+1} > \left(1+\frac{1}{n+1}\right)^{n+2}, \forall n \in N$.
4. $\left(1-\frac{1}{n}\right)^n < \left(1-\frac{1}{n+1}\right)^{n+1}, \forall n \in N$.
5. $(1+a)^n > 1+na+\frac{n(n-1)}{2}a^2, \ \forall n \in N : n \geq 3, \ a > 0$.
6. $(2n-1)! < n^{2n-1}, \ \forall n \in N : n \geq 2$.
7. $\sin^{2n} x + \cos^{2n} x \leq 1, \forall n \in N$.
8. Tengsizlikni isbotlang:

$$x_n = \underbrace{\sqrt{5+\sqrt{5+\ldots+.\sqrt{5}}}}_{n \quad ildiz} < \frac{1+\sqrt{21}}{2}, \quad \forall n \in N$$

References:

1. Abduraxmonov B.A. Matematik induksiya metodi. Toshkent. 2008.
2. Abduraxmonov B.A., Xakimova S.A., To'ychiyeva N.T. Matematik induksiya metodi yordamida tenglik va ayniyatlarni isbotlash// Eurasian journal of mathematical theory and computer sciences. Doi/10.5281/zenodo.6320503. 2022.
3. Соловьев Ю. П. Задачи по алгебре и теории чисел для математических школ. Ч. 1 - 3. — М.: школа им. А. Н. Колмогорова, 1998.
4. Соминский И. С. Метод математической индукции. Серия «популярные лекции по математике» — Вып. 3. — М.: Наука, 1974.
5. Бендукидзе А., Сулаквелидзе А. Вычисление сумм // Квант, № 9. 1970.