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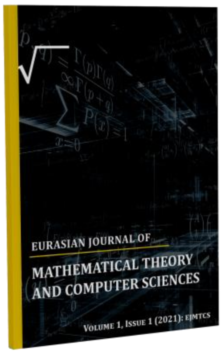


# USING THE VISCOSITY PARAMETER ON THE NAVIER-STOKES EQUATIONS FOR A VISCOUS INCOMPRESSIBLE FLUID IN AN UNBOUNDED DOMAIN

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**ABSTRACT**

*The solution of the Navier-Stokes equations for a viscous incompressible fluid in an unbounded domain has significant applications in the field of medicine, in particular, for modeling blood flow in the heart and blood vessels. In this context, the annotation can be described as follows: This work is devoted to the study of solutions of the Navier-Stokes equations for a viscous incompressible fluid in an unlimited region with application to the modeling of blood flow in the heart and blood vessels. The main attention is paid to the development of mathematical models and numerical methods for solving this problem, as well as the analysis of the results obtained.*

## INTRODUCTION

Blood viscosity plays an important role in the cardiovascular system. High viscosity can lead to a complication of blood flow, an increase in frictional forces between the walls of blood vessels, which increases the likelihood of thrombosis and atherosclerosis. The viscosity of blood depends on its composition and properties of red blood cells. In addition, viscosity greatly affects the work of the heart. With a decrease in blood viscosity, the resistance to veins and veins decreases, which reduces the work of the heart. With an increase in blood viscosity, more heart effort is needed to pump it through the blood vessels. Thus, optimal blood viscosity is a prerequisite for the normal functioning of the cardiovascular system. Viscosity is the friction force between molecules that prevents the free flow of liquids. In vessels, viscosity plays an important role in transporting blood through the circulatory system. Blood consists of cells and plasma, which contains proteins, sugars, fats and other substances. When blood moves through the vessels, it contacts the inner surface of the vessel walls, which is smooth and slightly covered with mucus. This facilitates the movement of blood inside the vessels, but the viscosity still affects the movement of blood.

Increased viscosity in the blood can be caused by certain medical conditions, such as elevated blood cell levels, elevated levels of fats and other substances. This can lead to the formation of blood clots and even to a heart attack or stroke. On the other hand, low blood viscosity can be caused by certain conditions, such as elevated blood water levels or low blood cell counts. This can lead to the appearance of hematomas and bleeding.

Given that blood viscosity is an important factor in the pathophysiology of many diseases, determining its level and monitoring is an important tool for the diagnosis and treatment of these conditions.

Several well-known scientists who have studied viscosity include:

Isaac Newton - created a theory about the viscosity of liquids.

Einstein - studied the viscosity and diffusion of molecules in various materials.

Jean Baptiste Bordeaux - studied the viscosity of water and other liquids.

Oswald Rippe - studied the viscosity of polymers.

Charles Coles - studied the viscosity of liquids at high pressures.

Ludwig Boltzmann - created a theory about the viscosity of gases.

John Hurt: English physician and physiologist who studied blood pressure and blood flow.

Richard Preston: Professor of Physics at the University of Cambridge, who has done research in colloid physics and biophysics.

Gaito File: A German physiologist who in 1921 first used the term "blood" to describe the phenomenon under study.

Andrew Vine: An American physiologist who has researched the role of blood in the cardiovascular system and related problems.

William Fife: A Scottish physician who in 1827 published the results of research on the theory of blood viscosity.

Leonard Azer: a well-known biophysicist who has made a significant contribution to the study of the mechanisms of moving fluids in organisms.

Germanoz Izdrug: a French physicist and biologist who worked in the field of medicine and biophysics, who studied the properties of blood and antibodies.

In general, the normal content of proteins and other substances in the blood is important for maintaining optimal blood viscosity and its free flow through the vessels. Doctors can perform blood tests to assess its viscosity and manage the risk of various medical problems. Viscosity is one of the key parameters affecting hemodynamics. It determines the resistance of the blood as it moves through the vessels. There are several ways to display viscosity in hemodynamics:

Measurement of rheological properties of blood. To do this, special devices are used, such as rheometers, which allow you to determine the dynamic and kinematic viscosity of blood.

Assessment of blood flow through a physical model of vessels using mathematical modeling methods. This makes it possible to determine the characteristics of hemodynamics in conditions of altered blood viscosity.

Indirect indicators – for example, an increase in fatty acids, which lead to a change in blood viscosity.

Determination of protein concentration. The concentration of proteins in the blood can decrease or increase in various pathological conditions, which, in turn, can affect viscosity and, consequently, hemodynamics. The part of the equation of fluid motion is a system of three non-homogeneous parabolic equations corresponding to three projections of fluid velocity, and the second part of the equation of fluid motion contains components of convective acceleration due to the inhomogeneity of the velocity field, the intensity of the field of mass forces and the pressure gradient.

**NAVIER-STOKES EQUATIONS FOR A VISCOUS INCOMPRESSIBLE FLUID**

The state of a moving fluid is determined by setting five values: three components of velocity *V(x; y; z; t)* pressure *p*(*x; y; z; t*) and density _(*x; y; z; t*) In fluid mechanics, its molecular structure is not considered, it is assumed that the fluid fills the space entirely, instead of the fluid itself, its model is studied, a fictitious continuous medium with the property of continuity. This approach simplifies the researching, all mechanical and hemodynamics characteristics of the liquid medium (velocity, pressure, density) are assumed to be continuous and differentiable.

The equations of motion of a viscous incompressible fluid (Navier-Stokes equations) in projections on the coordinate axis by velocity components have the form [1]

$$\frac{\partial \upsilon_x}{\partial t} = F_x - \frac{1}{\rho}\frac{\partial p}{\partial x} - \upsilon_x \frac{\partial \upsilon_x}{\partial x} - \upsilon_y \frac{\partial \upsilon_x}{\partial y} - \upsilon_z \frac{\partial \upsilon_x}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_x}{\partial x^2} + \frac{\partial^2 \upsilon_x}{\partial y^2} + \frac{\partial^2 \upsilon_x}{\partial z^2}\right), \quad (1)$$

$$\frac{\partial \upsilon_y}{\partial t} = F_y - \frac{1}{\rho}\frac{\partial p}{\partial y} - \upsilon_x \frac{\partial \upsilon_y}{\partial x} - \upsilon_y \frac{\partial \upsilon_y}{\partial y} - \upsilon_z \frac{\partial \upsilon_y}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_y}{\partial x^2} + \frac{\partial^2 \upsilon_y}{\partial y^2} + \frac{\partial^2 \upsilon_y}{\partial z^2}\right), \quad (2)$$

$$\frac{\partial \upsilon_z}{\partial t} = F_z - \frac{1}{\rho}\frac{\partial p}{\partial z} - \upsilon_x \frac{\partial \upsilon_z}{\partial x} - \upsilon_y \frac{\partial \upsilon_z}{\partial y} - \upsilon_z \frac{\partial \upsilon_z}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_z}{\partial x^2} + \frac{\partial^2 \upsilon_z}{\partial y^2} + \frac{\partial^2 \upsilon_z}{\partial z^2}\right), \quad (3)$$

where $\mathbf{V}(x,y,z,t) = \upsilon_x(x,y,z,t)\cdot\mathbf{i} + \upsilon_y(x,y,z,t)\cdot\mathbf{j} + \upsilon_z(x,y,z,t)\cdot\mathbf{k},$; values of x; y; z; t – are called Euler variables, $\mathbf{F}(x,y,z,t) = F_x(x,y,z,t)\mathbf{i} + F_y(x,y,z,t)\mathbf{j} + F_z(x,y,z,t)\mathbf{k}$ -is the intensity of the field of mass forces *grad* $p = \nabla p = \frac{\partial p}{\partial x}\mathbf{i} + \frac{\partial p}{\partial y}\mathbf{j}\frac{\partial p}{\partial z}\mathbf{k},$ $p = p(x,y,z),$

$\nabla$ – operator "nabla" , $\rho$ – density of the liquid, υ– kinematic viscosity of the liquid,
*i, j, k* – orts.

The continuity equation for an incompressible fluid $\frac{dy}{dt} = 0$:

$$div\mathbf{V} = \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} + \frac{\partial \upsilon_z}{\partial z} = 0, \quad \forall(x,y,z), \forall t \quad (4)$$

The main task of hydrodynamics is to find the following functions of coordinates and time: $\upsilon_x = f_1(x,y,z,t)$

$\upsilon_y = f_2(x,y,z,t)$, $\upsilon_z = f_3(x,y,z,t)$, $p = f_4(x,y,z,t)$ under the given initial conditions $(\rho = const > 0)$. (5)

$\upsilon_x|_{t=0} = f_1(x,y,z,0)$, $\upsilon_y|_{t=0} = f_2(x,y,z,0)$, $\upsilon_z|_{t=0} = f_3(x,y,z,0)$ under the given initial conditions. (6)

The equations of motion of a viscous incompressible fluid (1)-(4), tested in practice, adequately reflect the physical phenomenon in liquids and are a correct mathematical model. Therefore, the equations of motion (1)-(3) and continuity (4) are sufficient to solve the main problem of hydrodynamics when $v_x(x;\ y;\ z;\ t)$; $v_y(x;\ y;\ z;\ t)$;$v_z(x;\ y;\ z;\ t)$ – continuously differentiable functions with respect to t and twice continuously differentiable functions with respect to variables *x; y; z*; in the domain $(x,y,z)\in\Omega=R^3$, $t\in T=\{t\in R^1/t>0\}$.

$\upsilon_x(x,y,z,t)\in C^{2,2,2,1}_{x,y,z,t}(\Omega\times T)$, $\upsilon_y(x,y,z,t)\in C^{2,2,2,1}_{x,y,z,t}(\Omega\times T)$, $\upsilon_z(x,y,z,t)\in C^{2,2,2,1}_{x,y,z,t}(\Omega\times T)$

We assume that $F_x(x,y,z,t)$, $F_y(x,y,z,t)$, $F_z(x,y,z,t)$ are given continuous functions in the domain of $\Omega\times T$.

$F_x(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $F_y(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $F_z(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$ and functions $p_x(x,y,z,t)=\dfrac{\partial\rho}{\partial x}\in C_{x,y,z,t}$, $p_y(x,y,z,t)=\dfrac{\partial\rho}{\partial y}\in C_{x,y,z,t}$, $p_z(x,y,z,t)=\dfrac{\partial\rho}{\partial z}\in C_{x,y,z,t}$.

In the classical formulation, the initial problem for the Navier-Stokes equations for a viscous incompressible fluid in an unbounded domain $\Omega\times T$ has the form: find functions, $\upsilon_x(x,y,z,t):\Omega\times T\to R^1$, $\upsilon_y(x,y,z,t):\Omega\times T\to R^1$, $\upsilon_z(x,y,z,t):\Omega\times T\to R^1$

such that they satisfy equations (1)-(3) in Ω×T and the continuity equation (4) under given initial conditions (6), where $f_i(x,y,z,0)\in C(\Omega)$, $|f_i(x,y,z,0)|\le c_i$, $c_i=const>0$, $i=1,2,3$.

The proposed method for solving this problem is obtained on the basis of the author's works published in [5-7]. For simplicity of presentation of the results obtained, a regular solution of the Navier-Stokes equation is given below.

**2.1. Systems of parabolic equations**

Suppose that solutions of system (1)-(4) with initial condition (6) are known.

Then

$$\frac{\partial\upsilon_x}{\partial t}=\nu\left(\frac{\partial^2\upsilon_x}{\partial x^2}+\frac{\partial^2\upsilon_x}{\partial y^2}+\frac{\partial^2\upsilon_x}{\partial z^2}\right)+\omega(x,y,z,t)$$

$$\frac{\partial\upsilon_y}{\partial t}=\nu\left(\frac{\partial^2\upsilon_y}{\partial x^2}+\frac{\partial^2\upsilon_y}{\partial y^2}+\frac{\partial^2\upsilon_y}{\partial z^2}\right)+\mu(x,y,z,t)$$

$$\frac{\partial\upsilon_z}{\partial t}=\nu\left(\frac{\partial^2\upsilon_z}{\partial x^2}+\frac{\partial^2\upsilon_z}{\partial y^2}+\frac{\partial^2\upsilon_z}{\partial z^2}\right)+\theta(x,y,z,t)$$

Where

$$\omega(x,y,z,t)=F_x-\frac{1}{\rho}\frac{\partial p}{\partial x}-\upsilon_x\frac{\partial\upsilon_x}{\partial x}-\upsilon_y\frac{\partial\upsilon_x}{\partial y}-\upsilon_z\frac{\partial\upsilon_x}{\partial z}$$

$$\mu(x,y,z,t)=F_y-\frac{1}{\rho}\frac{\partial p}{\partial y}-\upsilon_x\frac{\partial\upsilon_y}{\partial x}-\upsilon_y\frac{\partial\upsilon_y}{\partial y}-\upsilon_z\frac{\partial\upsilon_y}{\partial z}$$

$$\theta(x,y,z,t)=F_z-\frac{1}{\rho}\frac{\partial p}{\partial z}-\upsilon_x\frac{\partial\upsilon_z}{\partial x}-\upsilon_y\frac{\partial\upsilon_z}{\partial y}-\upsilon_z\frac{\partial\upsilon_z}{\partial z}$$

in front of everything $(x,y,z)\in\Omega=R^3$, $t\in T$.

Let the function $U(x,y,z,t)\in C^{2,2,2,1}(\Omega\times T)$, $V(x,y,z,t)\in C^{2,2,2,1}(\Omega\times T)$, $W(x,y,z,t)\in C^{2,2,2,1}(\Omega\times T)$ – solutions of the following equations

$$\frac{\partial U}{\partial t}=\nu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right)+\omega_1(x,y,z,t)$$

$$\frac{\partial V}{\partial t}=\nu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)+\mu_1(x,y,z,t)$$

$$\frac{\partial W}{\partial t}=\nu\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}\right)+\theta_1(x,y,z,t)$$

with initial conditions

$U|_{t=0}=f_1(x,y,z,0)$, $V|_{t=0}=f_2(x,y,z,0)$, $W|_{t=0}=f_3(x,y,z,0)$,

in the area of $(x,y,z)\in\Omega=R^3$, $t\in T$

In here $\omega_1(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $\mu_1(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $\theta_1(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$

unknown bounded absolutely integral continuous functions – unknown bounded absolutely integral continuous functions. In particular if $\omega_1(x,y,z,t)=\omega(x,y,z,t)$, $\mu_1(x,y,z,t)=\mu(x,y,z,t)$, $\theta_1(x,y,z,t)=\theta(x,y,z,t)$ else $U(x,y,z,t)=\upsilon_x(x,y,z,t)$, $V(x,y,z,t)=\upsilon_y(x,y,z,t)$, $W(x,y,z,t)=\upsilon_z(x,y,z,t)$ in $(x,y,z)\in\Omega$, $t\in T$.

Systems of parabolic equations (13)-(15) with initial conditions (16) have solutions [8, 9]:

$$U(x,y,z,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{1}{\left(2\sqrt{\nu\pi t}\right)^3}f_1(\xi,\eta,\zeta)e^{-\frac{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2}{4\nu t}}d\xi d\eta d\zeta+$$

$$+\int_0^t\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\omega_1(\xi,\eta,\zeta,\tau)}{\left(2\sqrt{\pi\nu(t-\tau)}\right)^3}e^{-\frac{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2}{4\nu t}}d\xi d\eta d\zeta d\tau \quad (17)$$

$$V(x,y,z,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{f_2(\xi,\eta,\zeta)}{\left(2\sqrt{\nu\pi t}\right)^3}e^{-\frac{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2}{4\nu t}}d\xi d\eta d\zeta+$$

$$+\int_0^t\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\mu_1(\xi,\eta,\zeta,\tau)}{\left(2\sqrt{\pi\nu(t-\tau)}\right)^3}e^{-\frac{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2}{4\nu(t-\tau)}}d\xi d\eta d\zeta d\tau \quad (18)$$

$$W(x,y,z,t)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{f_3(\xi,\eta,\zeta)}{\left(2\sqrt{\nu\pi t}\right)^3}e^{-\frac{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2}{4\nu t}}d\xi d\eta d\zeta+$$

$$+\int_0^t\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{\theta_1(\xi,\eta,\zeta,\tau)}{\left(2\sqrt{\pi\nu(t-\tau)}\right)^3}e^{-\frac{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2}{4\nu(t-\tau)}}d\xi d\eta d\zeta d\tau \quad (19)$$

Introducing notation $G(x,y,z,t)=\frac{1}{(2\sqrt{\pi\nu t})^3}e^{-\frac{(\xi-x)^2+(\eta-y)^2+(\zeta-z)^2}{4\nu t}}$, $d\sigma=d\xi d\eta d\zeta$

solutions (17)-(19) are written as

$$U(x,y,z,t)=\int_\Omega G(x,y,z,t,\sigma)\,f_1(\sigma)d\sigma+\int_0^t\int_\Omega G(x,y,z,t-\tau,\sigma)\,\omega_1(\sigma,\tau)d\sigma d\tau \quad (20)$$

$$V(x,y,z,t)=\int_\Omega G(x,y,z,t,\sigma)\,f_2(\sigma)d\sigma+\int_0^t\int_\Omega G(x,y,z,t-\tau,\sigma)\,\mu_1(\sigma,\tau)d\sigma d\tau \quad (21)$$

$$W(x,y,z,t)=\int_\Omega G(x,y,z,t,\sigma)\,f_3(\sigma)d\sigma+\int_0^t\int_\Omega G(x,y,z,t-\tau,\sigma)\,\theta_1(\sigma,\tau)d\sigma d\tau \quad (22)$$

Where $\Omega=R^3$, $\sigma=(\xi,\eta,\zeta)$ we note that $\omega_1(\sigma,\tau)$, $\mu_1(\sigma,\tau)$, $\theta_1(\sigma,\tau)$ – unknown bounded absolutely integral continuous functions in $\tau\in\{0\le\tau<t, t\in T\}=T_1$

$\omega_1(\sigma,\tau)$, $\mu_1(\sigma,\tau)$, $\theta_1(\sigma,\tau)$, $\sigma\in\Omega$, $\tau\in T_1$

**SOLVING EQUATIONS**

**MATHEMATOCAL THEOREM.** The Navier-Stokes equations (1)-(3) for a viscous incompressible fluid under

any initial conditions (6) have solutions satisfying the continuity equation (4) in an unbounded domain if and only if the function $\Gamma(x,y,z,t)>0$ for any $(x,y,z)\in\Omega=R^3$; $t\in\mathrm{T}$; where Γ(x; y; z; t) is determined by the formula

$$\int_0^t\int_\Omega P(x,y,z,t-\tau,\sigma)\Lambda(\sigma,\tau)d\sigma d\tau=R(x,y,z,t)$$

Solutions $\upsilon_x(x,y,z,t)=S(x,y,z,t,\chi)$, $\upsilon_y(x,y,z,t)=\mathrm{T}(x,y,z,t,\chi)$, $\upsilon_z(x,y,z,t)=Q(x,y,z,t,\chi)$, $\upsilon_x(x,y,z,t)=S_0(x,y,z,t,\chi)$, $\upsilon_y(x,y,z,t)=\mathrm{T}_0(x,y,z,t,\chi)$, $\upsilon_z(x,y,z,t)=Q_0(x,y,z,t,\chi)$, in general, are not unique and are determined by formulas, respectively.

Solution $\upsilon_x(x,y,z,t)=S(x,y,z,t,\chi)$, $\upsilon_y(x,y,z,t)=\mathrm{T}(x,y,z,t,\chi)$, $\upsilon_z(x,y,z,t)=Q(x,y,z,t,\chi)$, $\upsilon_x(x,y,z,t)=S_0(x,y,z,t,\chi)$, $\upsilon_y(x,y,z,t)=\mathrm{T}_0(x,y,z,t,\chi)$, $\upsilon_z(x,y,z,t)=Q_0(x,y,z,t,\chi)$, the only one if

$$\int_0^t\int_\Omega \mathrm{P}_1(x,y,z,t-\tau,\sigma)\chi(\sigma,\tau)d\sigma d\tau \equiv 0, \qquad \int_0^t\int_\Omega \mathrm{P}_2(x,y,z,t-\tau,\sigma)\chi(\sigma,\tau)d\sigma d\tau \equiv 0$$

$$\int_0^t\int_\Omega \mathrm{P}_3(x,y,z,t,t-\tau,\sigma)\chi(\sigma,\tau)d\sigma d\tau \equiv 0, \quad \forall(x,y,z)\in\Omega=R^3,\ t\in\mathrm{T},$$

$\mathrm{P}_1,\mathrm{P}_2,\mathrm{P}_3$ where is

$S_0(x,y,z,t)=\alpha_1(x,y,z,t)$,

$\mathrm{T}_0(x,y,z,t)=\alpha_2(x,y,z,t)$,

$Q_0(x,y,z,t)=\alpha_3(x,y,z,t)$, defined by formulas

$$\pi(x,y,z,t-\tau,\sigma)=\begin{pmatrix}\mathrm{P}_1(x,y,z,t-\tau,\sigma)\\ \mathrm{P}_2(x,y,z,t-\tau,\sigma)\\ \mathrm{P}_3(x,y,z,t-\tau,\sigma)\end{pmatrix}$$

condition is written in the form of an integral equation with respect to $\chi(\sigma,\tau)$:

$$\int_0^t\int_\Omega \pi(x,y,z,t-\tau,\sigma)\chi(\sigma,\tau)d\sigma d\tau = 0$$

## IV. RESULTES

The results, which demonstrate the uniqueness of solutions of the Navier-Stokes equations for a viscous incompressible fluid under given initial conditions, as well as with the condition of continuity, are based on theoretical studies of mathematical hydrodynamics.

There is a theorem on the existence and uniqueness of solutions for the Navier-Stokes equations, which states that under given initial conditions and the condition of continuity, the Navier-Stokes equations have a unique solution with respect to the components of the fluid velocity. This means that any problem in hydrodynamics that is modeled by the Navier-Stokes equations has a unique solution.

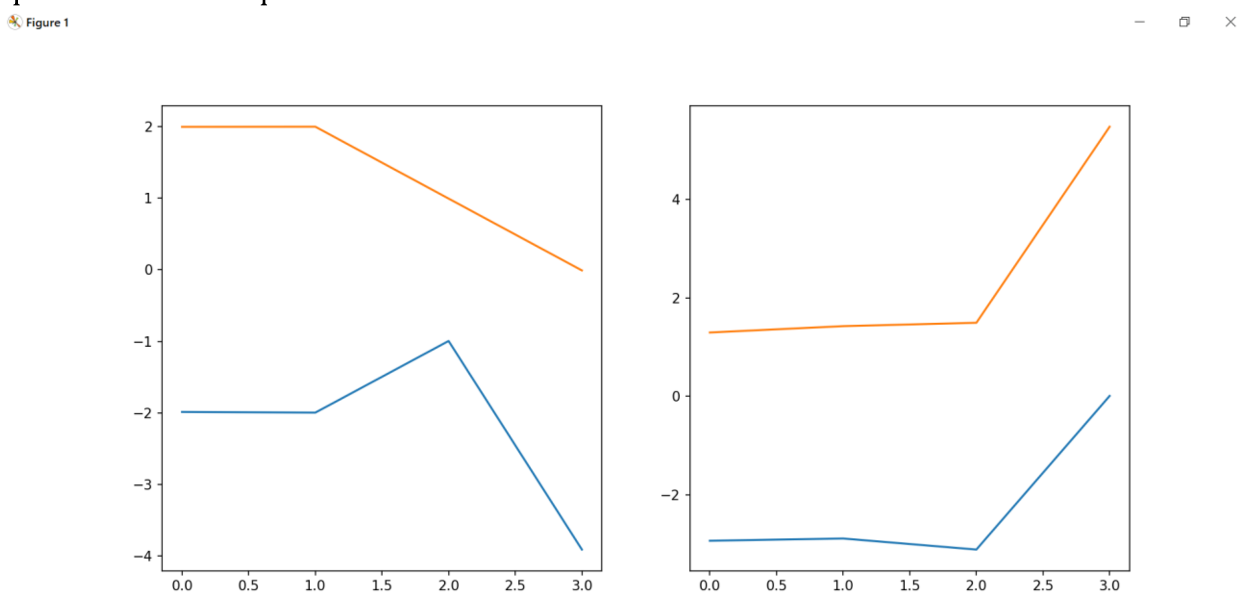


**Fig.1.** Estimate the range of velocity minimum and maximum

This theorem has important implications for many fields of science and technology, such as aerodynamics, geophysics, biomechanics and medicine. For example, in medicine, it can be used to simulate blood flow in the heart and blood vessels.

| Magnitude | Minimum | Maximum |
|---|---|---|
| **U:=x** | -1.99845 | 1.9992 |
| **V:=y** | -1.99727 | 1.99955 |
| **W:=z** | -0.995313 | 0.995265 |
| **P:** | -3.95736 | -0.0081647 |

**Table1.1**. Estimate the range of velocity

Although the existence and uniqueness theorem of solutions for the Navier-Stokes equations is an important result, in some cases it may be difficult to obtain an analytical solution to the equations. Therefore, numerical methods that allow solving the Navier-Stokes equations on a computer become important for practical application.

**CONCLUSION**

It is known that the Navier-Stokes equations for a viscous incompressible fluid under given initial conditions, as well as with the continuity condition, are among the most fundamental equations in hydrodynamics. An important issue is the existence and uniqueness of solutions to these equations with respect to the components of the fluid velocity. For this question, there is a theorem on the existence and uniqueness of solutions for the Navier-Stokes equations. According to this theorem, given the initial conditions and the continuity condition, the Navier-Stokes equations have a unique solution with respect to the components of the fluid velocity. However, this solution can be quite complex and difficult to obtain explicitly. Thus, the theorem on the existence and uniqueness of solutions to the Navier-Stokes equations is an important result for hydrodynamics and has wide application in various fields such as aerodynamics, geophysics, biomechanics, etc. In addition, this theorem is also of practical importance in the development of numerical methods for solving Navier-Stokes equations in real problems, such as modeling blood flow in the heart and blood vessels in medicine.

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