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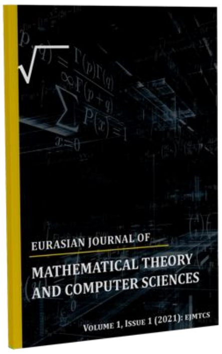

DESCRIPTION OF THE VISCOSITY PARAMETER IN THE EQUATIONS FOR A VISCOUS INCOMPRESSIBLE FLUID IN AN UNLIMITED REGION USING COMSOLE MULTYPYSICS

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ABSTRACT
The solution of the Navier-Stokes equations for a viscous incompressible fluid in an unbounded domain has important applications in medicine, particularly in simulating blood flow in the heart and veins. In this scenario, the annotation may be stated as follows: this research focuses on the solutions of the Navier-Stokes equations for a viscous incompressible fluid in an infinite area, with an application to the modeling of blood flow in the heart and blood arteries. The study describes several approaches for simulating blood flow, including hemodynamics in the aorta, major arteries, and minor vessels. Examples of blood flow velocity, pressure, and other characteristics are provided under a variety of settings, such as the treatment of cardiovascular disorders. In conclusion, this work makes an essential addition to the field of medical physics and hydrodynamics, and it can help scientists and clinicians investigate blood flow in diverse settings and create novel treatments for heart and vascular illnesses.

INTRODUCTION

Blood viscosity is an essential factor in the cardiovascular system. High viscosity can cause blood flow complications, such as increased frictional forces between blood vessel walls, increasing the risk of thrombosis and atherosclerosis. The viscosity of blood is determined by its composition and the qualities of red cells. Furthermore, viscosity has a significant impact on cardiac function. As blood viscosity falls, so does the resistance to veins and veins, reducing the heart's workload. As blood viscosity rises, the heart must work harder to push it through the blood arteries. Thus, appropriate cardiovascular system function requires adequate blood viscosity.

Viscosity is defined as the friction force between molecules that stops liquids from flowing freely. Viscosity in vessels is vital for moving blood throughout the cardiovascular system. Blood is composed of cells and plasma, which includes proteins, carbohydrates, lipids, and other things. When blood flows through the vessels, it comes into touch with the smooth, somewhat mucus-coated inner surface of the vessel walls. This improves blood mobility inside the vessels, yet viscosity still has an impact on blood flow. Certain medical problems, such as high blood cell counts, fat levels, and other chemicals, can induce increased blood viscosity. This can result in blood clots, as well as a heart attack or stroke.

Given that blood viscosity plays a significant role in the pathophysiology of many illnesses, assessing and monitoring its level is a valuable tool for diagnosis and therapy. Viscosity is a critical characteristic that influences hemodynamics. It regulates the resistance of blood as it flows through the arteries. There are numerous techniques to represent viscosity in hemodynamics:

- Measurement of blood rheology. To do this, specific machines such as rheometers are employed to determine the dynamic and kinematic viscosity of the blood.
- Calculation of blood flow through a physical model of vessels using mathematical modeling approaches. This allows us to assess hemodynamic properties under settings of changing blood viscosity.
- Indirect indicators – for example, an increase in fatty acids, which lead to a change in blood viscosity.
- Calculation of protein concentration. Protein concentrations in the blood can drop or rise under a variety of clinical circumstances, affecting viscosity and, as a result, hemodynamics. The first part of the fluid motion equation is a system of three non-homogeneous parabolic equations that correspond to three projections of fluid velocity, while the second part contains components of convective acceleration caused by the in homogeneity of the velocity field, the intensity of the field of mass forces, and the pressure gradient.

NAVIER-STOKES EQUATIONS FOR A VISCOUS INCOMPRESSIBLE FLUID

The state of a moving fluid is determined by setting five values: three components of velocity *V(x; y; z; t)* pressure *p*(*x; y; z; t*) and density _(*x; y; z; t*) In fluid mechanics, its molecular structure is not considered, it is assumed that the fluid fills the space entirely, instead of the fluid itself, its model is studied, a fictitious continuous medium with the property of continuity. This approach simplifies the researching, all mechanical and hemodynamics characteristics of the liquid medium (velocity, pressure, density) are assumed to be continuous and differentiable.

The equations of motion of a viscous incompressible fluid (Navier-Stokes equations) in projections on the coordinate axis by velocity components have the form [1,6]

$$\frac{\partial \upsilon_x}{\partial t} = F_x - \frac{1}{\rho}\frac{\partial p}{\partial x} - \upsilon_x \frac{\partial \upsilon_x}{\partial x} - \upsilon_y \frac{\partial \upsilon_x}{\partial y} - \upsilon_z \frac{\partial \upsilon_x}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_x}{\partial x^2} + \frac{\partial^2 \upsilon_x}{\partial y^2} + \frac{\partial^2 \upsilon_x}{\partial z^2}\right), \quad (1)$$

$$\frac{\partial \upsilon_y}{\partial t} = F_y - \frac{1}{\rho}\frac{\partial p}{\partial y} - \upsilon_x \frac{\partial \upsilon_y}{\partial x} - \upsilon_y \frac{\partial \upsilon_y}{\partial y} - \upsilon_z \frac{\partial \upsilon_y}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_y}{\partial x^2} + \frac{\partial^2 \upsilon_y}{\partial y^2} + \frac{\partial^2 \upsilon_y}{\partial z^2}\right), \quad (2)$$

$$\frac{\partial \upsilon_z}{\partial t} = F_z - \frac{1}{\rho}\frac{\partial p}{\partial z} - \upsilon_x \frac{\partial \upsilon_z}{\partial x} - \upsilon_y \frac{\partial \upsilon_z}{\partial y} - \upsilon_z \frac{\partial \upsilon_z}{\partial z} + \nu\left(\frac{\partial^2 \upsilon_z}{\partial x^2} + \frac{\partial^2 \upsilon_z}{\partial y^2} + \frac{\partial^2 \upsilon_z}{\partial z^2}\right), \quad (3)$$

where $\mathbf{V}(x,y,z,t)=\upsilon_x(x,y,z,t)\cdot\mathbf{i}+\upsilon_y(x,y,z,t)\cdot\mathbf{j}+\upsilon_z(x,y,z,t)\cdot\mathbf{k}$,; values of x; y; z; t – are called Euler variables, $\mathbf{F}(x,y,z,t)=F_x(x,y,z,t)\mathbf{i}+F_y(x,y,z,t)\mathbf{j}+F_z(x,y,z,t)\mathbf{k}$ -is the intensity of the field of mass forces *grad* $p=\nabla p=\frac{\partial p}{\partial x}\mathbf{i}+\frac{\partial p}{\partial y}\mathbf{j}\frac{\partial p}{\partial z}\mathbf{k}$, $p=p(x,y,z)$,

∇ – operator "nabla" , ρ – density of the liquid, υ– kinematic viscosity of the liquid,

i, j, k – orts. The continuity equation for an incompressible fluid $\frac{dy}{dt}=0$:

$$div\mathbf{V}=\frac{\partial\upsilon_x}{\partial x}+\frac{\partial\upsilon_y}{\partial y}+\frac{\partial\upsilon_z}{\partial z}=0,\ \forall(x,y,z),\ \forall t \quad (4)$$

The main task of hydrodynamics is to find the following functions of coordinates and time: $\upsilon_x=f_1(x,y,z,t)$

$\upsilon_y=f_2(x,y,z,t)$, $\upsilon_z=f_3(x,y,z,t)$, $p=f_4(x,y,z,t)$ under the given initial conditions $(\rho=const>0)$. (5)

$\upsilon_x|_{t=0}=f_1(x,y,z,0)$, $\upsilon_y|_{t=0}=f_2(x,y,z,0)$, $\upsilon_z|_{t=0}=f_3(x,y,z,0)$ under the given initial conditions. (6)

The equations of motion of a viscous incompressible fluid (1)-(4), tested in practice, adequately reflect the physical phenomenon in liquids and are a correct mathematical model. Therefore, the equations of motion (1)-(3) and continuity (4) are sufficient to solve the main problem of hydrodynamics when $v_x(x;\ y;\ z;\ t)$; $v_y(x;\ y;\ z;\ t)$;$v_z(x;\ y;\ z;\ t)$ – continuously differentiable functions with respect to t and twice continuously differentiable functions with respect to variables *x; y; z*; in the domain [4,6] $(x,y,z)\in\Omega=R^3$, $t\in T=\{t\in R^1/t>0\}$.

$\upsilon_x(x,y,z,t)\in C^{2,2,2,1}_{x,y,z,t}(\Omega\times T)$, $\upsilon_y(x,y,z,t)\in C^{2,2,2,1}_{x,y,z,t}(\Omega\times T)$, $\upsilon_z(x,y,z,t)\in C^{2,2,2,1}_{x,y,z,t}(\Omega\times T)$

We assume that $F_x(x,y,z,t)$, $F_y(x,y,z,t)$, $F_z(x,y,z,t)$ are given continuous functions in the domain of $\Omega\times T$.

$F_x(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $F_y(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $F_z(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$ and functions $p_x(x,y,z,t)=\frac{\partial\rho}{\partial x}\in C_{x,y,z,t}$, $p_y(x,y,z,t)=\frac{\partial\rho}{\partial y}\in C_{x,y,z,t}$, $p_z(x,y,z,t)=\frac{\partial\rho}{\partial z}\in C_{x,y,z,t}$.

In the classical formulation, the initial problem for the Navier-Stokes equations for a viscous incompressible fluid in an unbounded domain $\Omega\times T$ has the form: find functions, $\upsilon_x(x,y,z,t):\Omega\times T\to R^1$, $\upsilon_y(x,y,z,t):\Omega\times T\to R^1$, $\upsilon_z(x,y,z,t):\Omega\times T\to R^1$

such that they satisfy equations (1)-(3) in Ω×T and the continuity equation (4) under given initial conditions (6), where $f_i(x,y,z,0)\in C(\Omega)$, $|f_i(x,y,z,0)|\le c_i$, $c_i=const>0$, $i=1,2,3$.

The proposed method for solving this problem is obtained on the basis of the author's works published in [5-7]. For simplicity of presentation of the results obtained, a regular solution of the Navier-Stokes equation is given below[5,7].

Suppose that solutions of system (1)-(4) with initial condition (6) are known.
Then

$$\frac{\partial \upsilon_x}{\partial t}=\nu\left(\frac{\partial^2 \upsilon_x}{\partial x^2}+\frac{\partial^2 \upsilon_x}{\partial y^2}+\frac{\partial^2 \upsilon_x}{\partial z^2}\right)+\omega(x,y,z,t)$$

$$\frac{\partial \upsilon_y}{\partial t}=\nu\left(\frac{\partial^2 \upsilon_y}{\partial x^2}+\frac{\partial^2 \upsilon_y}{\partial y^2}+\frac{\partial^2 \upsilon_y}{\partial z^2}\right)+\mu(x,y,z,t)$$

$$\frac{\partial \upsilon_z}{\partial t}=\nu\left(\frac{\partial^2 \upsilon_z}{\partial x^2}+\frac{\partial^2 \upsilon_z}{\partial y^2}+\frac{\partial^2 \upsilon_z}{\partial z^2}\right)+\theta(x,y,z,t)$$

Where

$$\omega(x,y,z,t)=F_x-\frac{1}{\rho}\frac{\partial p}{\partial x}-\upsilon_x\frac{\partial \upsilon_x}{\partial x}-\upsilon_y\frac{\partial \upsilon_x}{\partial y}-\upsilon_z\frac{\partial \upsilon_x}{\partial z}$$

$$\mu(x,y,z,t)=F_y-\frac{1}{\rho}\frac{\partial p}{\partial y}-\upsilon_x\frac{\partial \upsilon_y}{\partial x}-\upsilon_y\frac{\partial \upsilon_y}{\partial y}-\upsilon_z\frac{\partial \upsilon_y}{\partial z}$$

$$\theta(x,y,z,t)=F_z-\frac{1}{\rho}\frac{\partial p}{\partial z}-\upsilon_x\frac{\partial \upsilon_z}{\partial x}-\upsilon_y\frac{\partial \upsilon_z}{\partial y}-\upsilon_z\frac{\partial \upsilon_z}{\partial z}$$

in front of everything $(x,y,z)\in\Omega=R^3$, $t\in T$.

Let the function $U(x,y,z,t)\in C^{2,2,2,1}(\Omega\times T)$, $V(x,y,z,t)\in C^{2,2,2,1}(\Omega\times T)$, $W(x,y,z,t)\in C^{2,2,2,1}(\Omega\times T)$ – solutions of the following equations

$$\frac{\partial U}{\partial t}=\nu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}\right)+\omega_1(x,y,z,t)$$

$$\frac{\partial V}{\partial t}=\nu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}\right)+\mu_1(x,y,z,t)$$

$$\frac{\partial W}{\partial t}=\nu\left(\frac{\partial^2 W}{\partial x^2}+\frac{\partial^2 W}{\partial y^2}+\frac{\partial^2 W}{\partial z^2}\right)+\theta_1(x,y,z,t) \qquad (8)$$

with initial conditions

$U|_{t=0}=f_1(x,y,z,0)$, $V|_{t=0}=f_2(x,y,z,0)$ $\quad W|_{t=0}=f_3(x,y,z,0)$,

in the area of $(x,y,z)\in\Omega=R^3$, $t\in T$

In here $\omega_1(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $\mu_1(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$, $\theta_1(x,y,z,t)\in C_{x,y,z,t}(\Omega\times T)$ – unknown bounded absolutely integral continuous functions – unknown bounded absolutely integral continuous functions. In particular if $\omega_1(x,y,z,t)=\omega(x,y,z,t)$, $\mu_1(x,y,z,t)=\mu(x,y,z,t)$, $\theta_1(x,y,z,t)=\theta(x,y,z,t)$ else $U(x,y,z,t)=\upsilon_x(x,y,z,t)$, $V(x,y,z,t)=\upsilon_y(x,y,z,t)$, $W(x,y,z,t)=\upsilon_z(x,y,z,t)$ in (8) $(x,y,z)\in\Omega$, $t\in T$.

Systems of parabolic equations (13)-(15) with initial conditions (16) have solutions [8, 9].

IV. RESULTS

The results, which illustrate the uniqueness of Navier-Stokes equation solutions for a viscous incompressible fluid under specific beginning circumstances and with the continuity condition, are based on mathematical hydrodynamics theory. A theorem on the existence and uniqueness of solutions for the Navier-Stokes equations asserts that under specified beginning circumstances and continuity, the Navier-Stokes equations have a unique solution with regard to the fluid velocity components. This implies that each hydrodynamic issue handled by the Navier-Stokes equations has a unique solution [5,7].

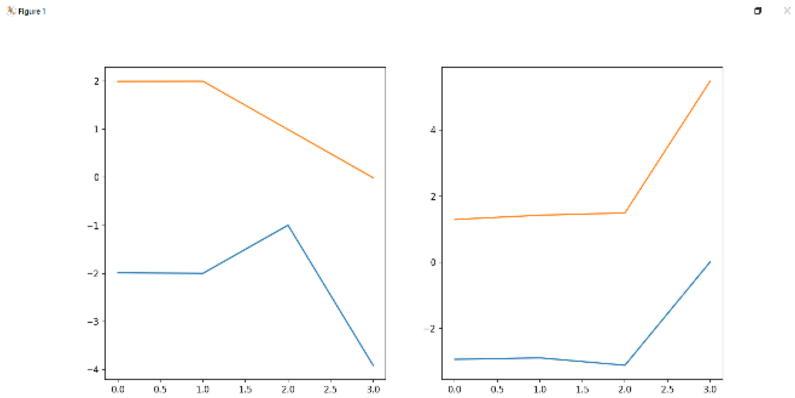

Fig.1. Estimate the range of velocity minimum and maximum

This theorem has important implications for many fields of science and technology, such as aerodynamics, geophysics, biomechanics and medicine. For example, in medicine, it can be used to simulate blood flow in the heart and blood vessels.

Magnitude	Minimum	Maximum
U:=x	-1.99845	1.9992
V:=y	-1.99727	1.99955
W:=z	-0.995313	0.995265
P:	-3.95736	-0.0081647

Table1. Estimate the range of velocity

Although the existence and uniqueness theorem for solutions to the Navier-Stokes equations is a significant finding, obtaining an analytical solution to the equations may be challenging in some instances. As a result, numerical approaches for solving the Navier-Stokes equations on a computer are becoming increasingly relevant in practical applications.

Furthermore, the diffusion equation, additional parabolic differential equations may be used to represent blood arteries, such as those that explain the propagation of electrical or chemical impulses in the nervous system or the propagation of heat in tissue during laser therapy. The particular equation used is determined by the system's features and the physical processes that must be represented.

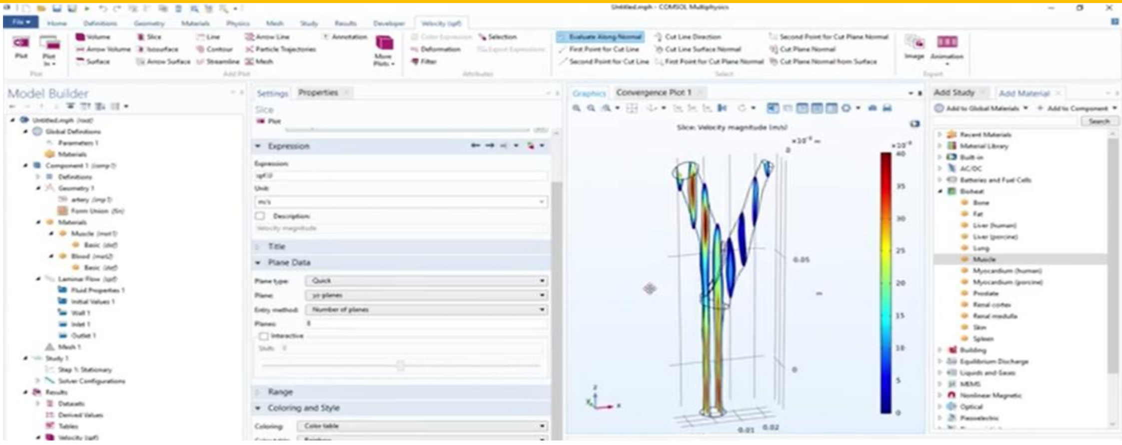

Fig 1. 1.Model of arteries velocity blood flow with oriented place

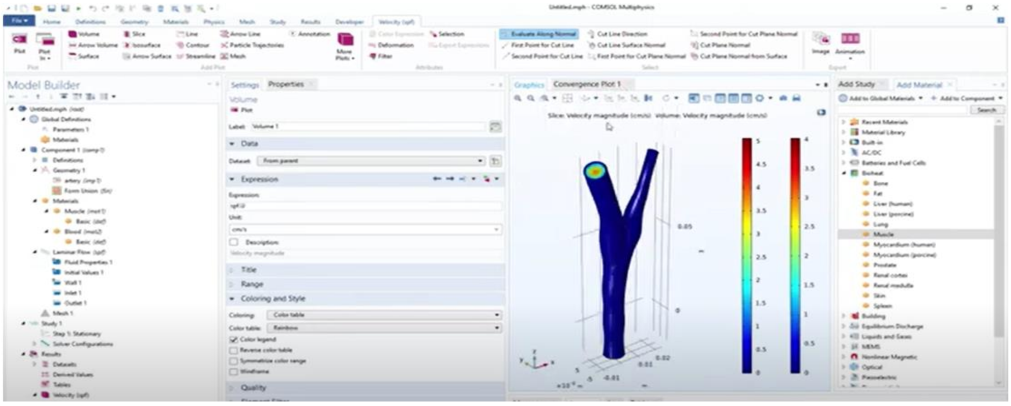

Fig 1.2. Model of arteries velocity blood flow with oriented place

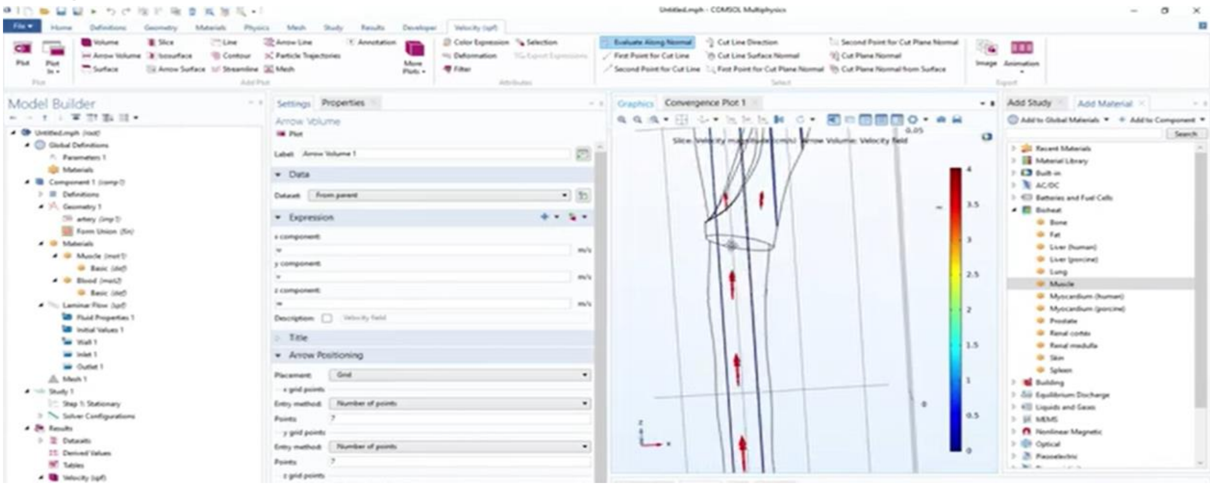

Fig1. 3. Model of arteries velocity blood flow with oriented place

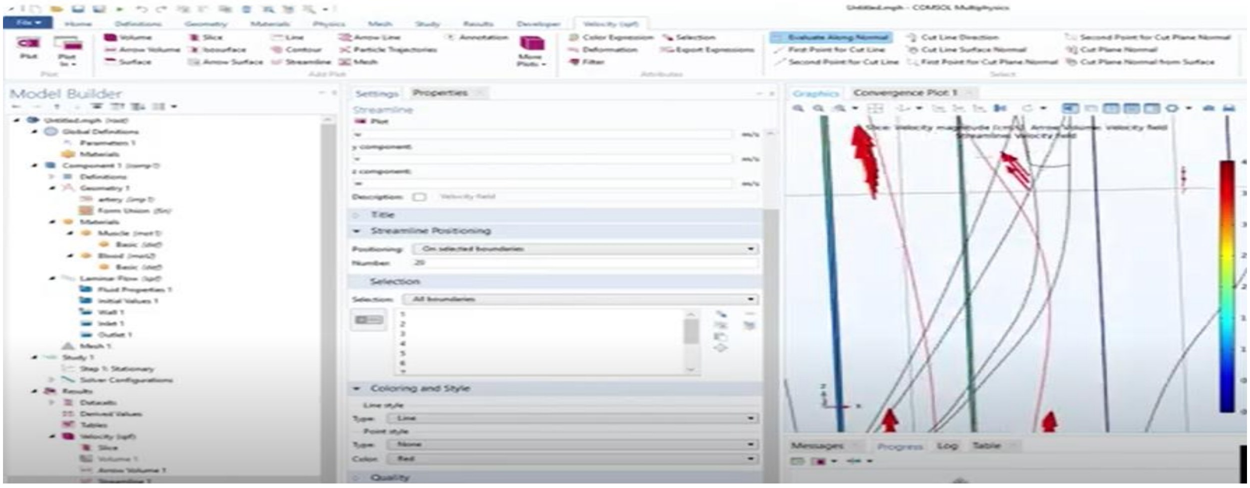

Fig 1.4. Model of arteries velocity blood flow with oriented place

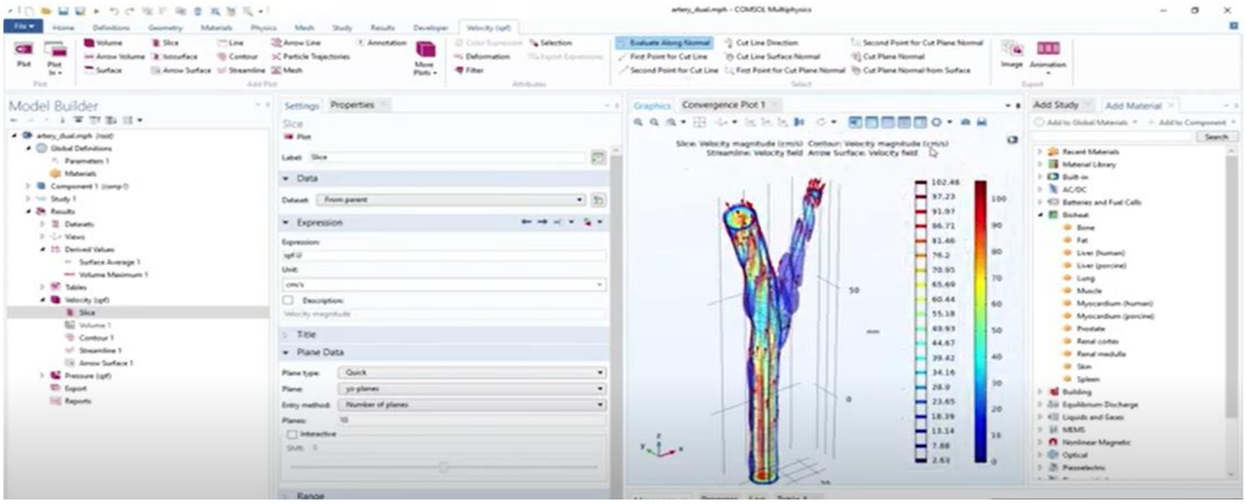

Fig1.5. Model of arteries velocity blood flow with oriented place

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