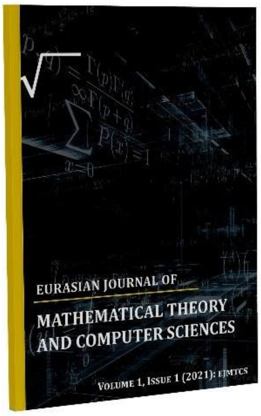
EURASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES
Innovative Academy Research Support Center
UIF = 8.3 | SJIF = 5.916 www.in-academy.uz



MATEMATIKA DARSLARIDA AYRIM TRIGONOMETRIK TENGSIZLIKLARNI ISBOTLASHNING NOAN'ANAVIY USULLARI

Xudayberganov G'ayrat[1], Bekchanov Sardor [2]
[1] UrDU "BTM" kafedrasi katta o'qituvchisi
[2] UrDU "BTM" kafedrasi o'qituvchisi.
https://doi.org/10.5281/zenodo.6596296

MAQOLA TARIXI
Qabul qilindi: 10-may 2022
Ma'qullandi: 14- may 2022
Chop etildi: 30- may 2022

ANNOTATSIYA
Ushbu maqolada matematika darslarida ayrim trigonometrik tengsizliklarni noan'anaviy usullar orqali isbotlash haqida mulohaza yuritilgan.

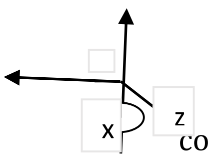
KALIT SO'ZLAR
Trigonometriya, vektor, tengsizlik, tenglik, ikki vektor orasidagi burchak.

Bugungi kunda o'quvchilar matematik masalalarni isbotlashda ko'proq tengsizliklarni isbot qilishga qiynalishadi. Bunga sabab tengsizliklar uchun asosan bir qolipga tushadigan isbotlash metodlari deyarli yo'qligidadir. Biz ushbu maqolada trigonometrik tengsizliklar uchun malum bir qoidaga tushadigan bazi tengsizliklarni vektorlar yordamida isbot qilishni o'rganamiz. Xususan yordamchi vektorlar yordamida almashtrishlar kiritgan holda tengsizliklarni soddaroq usullar bilan isbot qilish mumkinligini ko'rib chiqamiz. Bizga $\bar{a}$; $\bar{b}$; $\bar{c}$ vektorlar berilgan bo'lsin :

1. $(\bar{a},\bar{b}) = |\bar{a}| * |\bar{b}| * \cos(\bar{a}\wedge\bar{b})$
2. $(\bar{a}+\bar{b},\bar{c}) = (\bar{a},\bar{c}) + (\bar{b},\bar{c})$

Endi quyidagicha shartni kiritamiz $|\bar{a}| = |\bar{b}| = |\bar{c}| = 1$

$x+y+z=2\pi$



$(\bar{a}+\bar{b}+\bar{c}, \bar{a}+\bar{b}+\bar{c}) = |\bar{a}+\bar{b}+\bar{c}|^2 \geq 0$

$(\bar{a}+\bar{b}+\bar{c}, \bar{a}+\bar{b}+\bar{c}) = |\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2+2|\bar{a}| * |\bar{b}|\cos z + 2|\bar{a}| * |\bar{c}|\cos y + 2|\bar{b}| * |\bar{c}|\cos x$

Bu tengsizlikdan $\cos x + \cos y + \cos z \geq -\frac{3}{2}$, $x+y+z=2\pi$ ga teng.

Uchburchakda esa $\alpha+\beta+\gamma=\pi$ ga teng bo'ladi. Demak quyidagi almashtrish olamiz $x=2\alpha$, $y=2\beta$, $z=2\gamma$ Bu almashtrishdan $\cos 2\alpha+\cos 2\beta+\cos 2\gamma \geq -\frac{3}{2}$ tengsizlik hosil bo'ladi.

Endi 2α, 2β, 2γ dan qanday qilib, α, β, γ larga o'tamiz shuni ko'rib chiqamiz.

$\cos 2\alpha=2\cos\alpha^2-1$, $\cos 2\beta=2\cos\beta^2-1$, $\cos 2\gamma=2\cos\gamma^2-1$

$\cos 2\alpha+\cos 2\beta+\cos 2\gamma = 2(\cos\alpha^2+\cos\beta^2+\cos\gamma^2)-3$ ga teng bo'ladi.

$\cos 2\alpha+\cos 2\beta+\cos 2\gamma=2(\cos \alpha^2+\cos \beta^2+\cos \gamma^2)-3\geq -\frac{3}{2}$ bundan $\cos \alpha^2+\cos \beta^2+\cos \gamma^2 \geq \frac{3}{4}$ tengsizlik hosil bo'ladi.

$\cos \alpha^2=1-\sin \alpha^2$ $\cos \beta^2=1-\sin \beta^2$ $\cos \gamma^2=1-\sin \gamma^2$ ifodani o'rniga qo'ysak $\sin \alpha^2+\sin \beta^2+\sin \gamma^2 \leq \frac{9}{4}$ tengsizlik hosil bo'ladi. Koshi -Bunyakovskiy tengsizlikidan foydalanib yani

$(1+1+1)(a^2+b^2+c^2)\geq (a+b+c)^2$

$(1+1+1)(\sin \alpha^2+\sin \beta^2+\sin \gamma^2)\geq (\sin\alpha+\sin\beta+\sin\gamma)^2 \rightarrow$

$\sin\alpha+\sin\beta+\sin\gamma \leq \frac{3\sqrt{3}}{2}$ tengsizlikkga ega bo'lamiz.

Endi trigonometrik tenglikdan foydalanib quydagilarni isbot qilamiz.

$\sin\alpha+\sin\beta+\sin\gamma=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$

$4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} \leq \frac{3\sqrt{3}}{2}$

$\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} \leq \frac{3\sqrt{3}}{8}$ bo'ladi,

$\cos 2\alpha+\cos 2\beta+\cos 2\gamma=-1-4\cos\alpha\cos\beta\cos\gamma$ bundan

$\cos\alpha\cos\beta\cos\gamma \leq \frac{1}{8}$ bo'ladi, bundan

$\cos\alpha+\cos\beta+\cos\gamma$ uchun tengsizlik hosil qilamiz. Buning uchun $x=\pi-\alpha$, $y=\pi-\beta$, $z=\pi-\gamma$ deb tanlaymiz, natijada bularning yig'indisi $x+y+z=3\pi-(\alpha+\beta+\gamma)$ ga teng bo'ladi. Bu almashtrishdan esa quyidagi $\cos\alpha+\cos\beta+\cos\gamma \leq \frac{3}{2}$ tengsizlikka ega bo'lamiz. Bundan $\cos\frac{\alpha^2}{2}+\cos\frac{\beta^2}{2}+\cos\frac{\gamma^2}{2} \leq \frac{9}{4}$ bo'ladi.

Endi Koshi tengsizlikidan foydalansak

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}$$

$\frac{3\sqrt{3}}{2} \geq \cos\frac{\alpha}{2}+\cos\frac{\beta}{2}+\cos\frac{\gamma}{2} \geq 3\sqrt[3]{\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}} \rightarrow \cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2} \leq \frac{3\sqrt{3}}{8}$

hosil bo'ladi. Biz yuqoridagi tengsizliklarni isbotlash jarayonida quyidagi trigonometrik tengliklardan foydalandik.

1. $\sin\alpha+\sin\beta+\sin\gamma=4\cos\frac{\alpha}{2}\cos\frac{\beta}{2}\cos\frac{\gamma}{2}$
2. $\cos\alpha+\cos\beta+\cos\gamma=1+4\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}$
3. $\sin 2\alpha+\sin 2\beta+\sin 2\gamma=4\sin\alpha\sin\beta\sin\gamma$
4. $\cos 2\alpha+\cos 2\beta+\cos 2\gamma=-1-4\cos\alpha\cos\beta\cos\gamma$
5. $\mathrm{tg}\frac{\alpha}{2}\mathrm{tg}\frac{\beta}{2}+\mathrm{tg}\frac{\beta}{2}\mathrm{tg}\frac{\gamma}{2}+\mathrm{tg}\frac{\gamma}{2}\mathrm{tg}\frac{\alpha}{2}=1$
6. $\mathrm{ctg}\frac{\alpha}{2}\mathrm{ctg}\frac{\beta}{2}+\mathrm{ctg}\frac{\beta}{2}\mathrm{ctg}\frac{\gamma}{2}+\mathrm{ctg}\frac{\gamma}{2}\mathrm{ctg}\frac{\alpha}{2}=1$
7. $\sin\frac{\alpha^2}{2}+\sin\frac{\beta^2}{2}+\sin\frac{\gamma^2}{2}+2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\sin\frac{\gamma}{2}=1$
8. $\cos\alpha^2+\cos\beta^2+\cos\gamma^2+2\cos\alpha\cos\beta\cos\gamma=1$

Bu tengliklarni isbotlashni o'quvchilarga mustaqil topshiriq sifatida tavsiya qilishimiz mumkin.

FOYDALANILGAN ADABIYOTLAR

1. Mirzaahmedov.M , Sotiboldiyev T "Matematikadan olimpiada masalalari" Toshkent-2003 y
2. Гелфанд.F "методы доказательства неравенства" Москва-1972г
3. Сивяшинский.В "доказательство тригонометрических неравенств" Москва-1985г.