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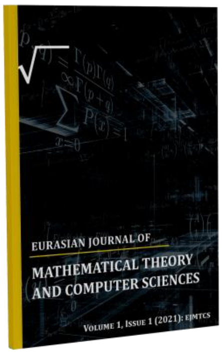


# IMPACT THEORY IN DRIVEN BOUNDARY PROBLEMS

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ABSTRACT
*In recent years, non-stationary issues of viscous-plastic body movement have attracted the attention of researchers [1-4, 8-9]. In the proposed work, the problem of impact of a solid body on a viscous-plastic rod of finite length is presented and an effective approximate solution is given. The issue of impact of a solid body on an adhesive-plastic rod was considered in the works of X. A. Mirzadjanzade [5], L. V. Nikitin [7].*

**INTRODUCTION.** Let a solid body moving with speed $V_0$ in the direction of the axis of the rod hit the fixed rod of finite length L made of incompressible viscous-plastic material at time t=0. (Figure 1)

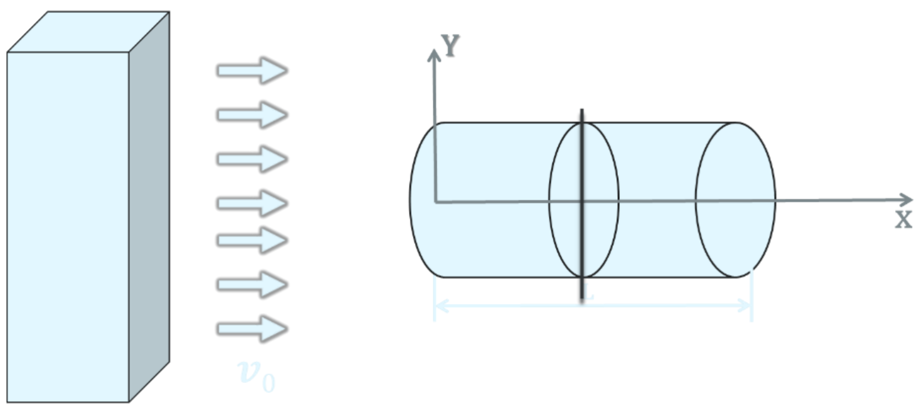


**Figure 1**

In this case, the motion is assumed to be quasi-one-dimensional, that is, the tension, speed, and other quantities are averaged over the stern cross-section.

In a darkening medium (speed dispersion), elastic excitations are infinitely large. It can be seen that in the 6u medium, the Young's modulus is infinitely large, and the excitation propagates throughout the entire rod. In this case, at an arbitrary moment of time t>0, the speed of movement is different from v0 at all points of the steering wheel. Therefore, depending on the movement of the points of the stern, the stern can be divided into two parts under the impact:

Part 1 – 0 < x < $x_0$(t) visco-plastic sphere. In this area, the stress modulus is greater than s0, and visco-plastic deformation is observed in this area.

Part 2 – $x_0$(t) < x < l a field that behaves like an absolute solid. In this area, the stress modulus is smaller than $\sigma_0$, and the movement of the rod in this area behaves like an absolute rigid body.

x=$x_0$(t) The dynamic boundary condition between the viscous-plastic and absolute solid-like domains is analyzed during the solution of the problem.

Therefore, the relationship between σ-stress and strain rate ∂u/∂x is introduced as:

$$\frac{\partial v}{\partial x}=\begin{cases}\dfrac{(\sigma+\sigma_0)}{\mu}, & (|\sigma|\geq\sigma_0)\\ 0, & (|\sigma|\leq\sigma_0)\end{cases}\qquad(1.1)$$

Here v(x,t) is the speed of the stern cross section at time "t",

"$\sigma_0$" is the ultimate stress, " μ " is the coefficient of viscosity of the stern material, the 0x coordinate is directed along the stern axis [2].

The equation of motion of the pivot points is as follows.:

$$\rho\frac{\partial v}{\partial t}=\frac{\partial\sigma}{\partial x}\qquad(1.2)$$

Here ρ is the density of the rod material, (constant)

(1.1) according to the relationship, the velocity in the 1st part of the stern, that is, in the visco-plastic region, satisfies the following diffusion equation:

$$\frac{\partial v}{\partial t}=a^2\frac{\partial^2 v}{\partial x^2},\qquad a^2=\frac{\mu}{\rho},\qquad 0<x<x_0(t),\qquad t>0\quad(1.3)$$

And the equation in the absolute solid sphere

$$\frac{\partial v}{\partial x}=0,\qquad x_0(t)<x<l,\qquad t>0\qquad(1.4)$$

appears.

From this $v_0=v_0(t)$ (where $v_0(t)$ the speed of movement of the points of the stern in the solid sphere (unknown)).

The equation of motion of the solid sphere according to Newton's 2nd law is:

$$M_c\frac{\partial v_0(t)}{\partial t}=\sigma(x_0(t)+0,t)F\qquad(1.5)$$

Here is $M_c$ the mass of the solid area of the stern, F is the surface of the cross section of the stern;

If we consider the continuity of voltage and velocity at this limit, $x_0=x_0(t)$

$$\begin{cases} v(x_0(t)+0,t) = -v_0(t) \\ \dfrac{\partial v(x_0(t),t)}{\partial x} = 0 \end{cases} \quad (1.6)$$

it follows that

Based $x_0 = x_0(t)$ on the equation (1.5) of stress continuity at the moving boundary, the following equation is derived.

$$\frac{\partial v_0(t)}{\partial t} = -\frac{\sigma_0}{\rho(l - x_0(t))}, \quad v_0(0) = 0, \quad x_0(t) < x < l \quad (1.7)$$

So, the following initial and boundary conditions were obtained for this problem:

$$\begin{cases} v(x,0) = 0, \quad 0 < x < l \\ \sigma(x,0) = 0, \quad 0 < x < l \\ M_T \dfrac{\partial v(0,t)}{\partial t} = F * \sigma(0,t), \quad (t > 0) \\ v(0,0) = V_0 \end{cases} \quad (1.8)$$

$$\begin{cases} v(x_0(t),t) = -v_0(t) \\ \dfrac{\partial v(x_0(t),t)}{\partial x} = 0 \end{cases} \quad (1.9)$$

**Solving the system of basic equations into dimensionless linear ordinary differential equations.** The problem of viscoplastic rod hitting an absolutely solid wall is reduced to solving the variable boundary diffusion equation to be found. Therefore, for convenience, all quantities in our equations are rendered dimensionless.

For this, the following definitions are introduced:

$$u(\xi,\tau) = \frac{v(x,t)}{V_0}, \quad \xi = \frac{x}{l}, \quad \xi_0(\tau) = \frac{x_0(t)}{l}, \quad u_0(\tau) = \frac{v_0(t)}{V_0}, \quad \sigma'(\xi,\tau) = \frac{\sigma(x,t)}{\sigma_0}, \quad \tau = \frac{a^2 * t}{l^2} \quad (2.1)$$

these designations (2.1), the following relations are formed to determine the unknown functions u(x, t), $x_0$ (t) and $u_0$ (t) in the system of equations (1.3), (1.7) - (1.9) :

$$\begin{cases} \dfrac{\partial u(\xi,\tau)}{\partial \tau} = \dfrac{\partial^2 u(\xi,\tau)}{\partial \xi^2}, \quad 0 < \xi < \xi_0(\tau) \\ \dfrac{du_0(\tau)}{d\tau} = -\dfrac{s}{1-\xi_0(\tau)}, \quad u_0(0) = 0, \quad \xi_0(\tau) < \xi < 1 \end{cases} \quad (2.2)$$

The boundary and initial conditions of the problem are as follows:

$$\begin{cases} u(\xi,0) = 0, \quad 0 < \xi < 1 \\ \sigma'(\xi,0) = 0, \quad 0 < \xi < 1 \\ \dfrac{\partial u(0,\tau)}{\partial \tau} = -ms\sigma'(0,\tau), \quad s = \dfrac{\sigma_0 l}{\mu u_0}, \quad (\tau > 0) \\ u(0,0) = 1 \end{cases} \quad (2.4)$$

$$\begin{cases} u(\xi_0(\tau),\tau) = -u_0(\tau) \\ \dfrac{\partial u(\xi_0(\tau),\tau)}{\partial \xi} = 0 \end{cases} \quad (2.3)$$

Here $s = \dfrac{\sigma_0 l}{\mu u_0}$ is the Sen-Venan parameter, which is a dimensionless combination of the limiting value of the stress, the rod length, the dynamic viscosity coefficient, and the initial velocity-dependent parameters ( $m = \dfrac{M_c}{M_T}$ - mutual ratio of sturgeon and absolute solids).

Thus, through the initial and boundary conditions of equations (2.2) (2.3)-(2.4), u(x, t), $x_0$ (t) and $u_0$ (t) can be found.

To solve the above system, according to the Karman-Polhausen method [6] used in the theory of the boundary layer, the function u(x, t) is searched in the following form:

$$u(\xi,\tau)=\begin{cases} a_0+a_1\dfrac{\xi}{\xi_0(\tau)}+a_2\dfrac{\xi^2}{\xi_0^{\,2}(\tau)}, & 0<\xi<\xi_0(\tau) \\ -u_0(\tau), & \xi_0(\tau)<\xi<1 \end{cases} \qquad (2.5)$$

Here the unknowns $a_i$ ($i$ = 0,1,2) and $x_0$ (t) are functions of t, that is, $a_i = a_i$ (t) and $x_0 = x_0$ (t).

If the functions $u_0$ (t) and $x_0$ (t) are respectively $u_0$ (0)=0 and $x_0$ (0)= 0, then the solution (2.5) does not satisfy equation (2.2). Therefore, it is required that the solution (2.5) satisfies the integral relation, which is derived from the integration of the equation (2.2) over the visco-plastic field:

$$\int_0^{\xi_0(\tau)} Lu\,d\xi = 0, \qquad Lu = \frac{\partial u(\xi,\tau)}{\partial\tau} - \frac{\partial^2 u(\xi,\tau)}{\partial\xi^2} \qquad (2.6)$$

To satisfy these relations, the unknown functions a $_i$ (t) in (2.5) are found using conditions (2.3).

$$\begin{cases} a_1 = -2(u_0(\tau)+a_0(\tau)) \\ a_2 = u_0(\tau)+a_0(\tau) \end{cases} \qquad (2.7)$$

$$\frac{da_0(\tau)}{d\tau} = m\Big(\frac{a_1(\tau)}{\xi_0(\tau)} - s\Big) \qquad (2.8)$$

After that, if the integral (2.6) is written in the following form:

$$\int_0^{\xi_0(\tau)} \frac{\partial u(\xi_0(\tau),\tau)}{\partial\tau}\,d\xi = \int_0^{\xi_0(\tau)} \frac{\partial^2 u(\xi_0(\tau),\tau)}{\partial\xi^2}\,d\xi \qquad (2.9)$$

When this (2.9) is integrated piecewise on the left and right side of Equation, taking into account that, the following relation is obtained:

$$\frac{d\xi^2{}_0(\tau)}{d\tau} = \frac{12(u_0(\tau)+a_0(\tau)) + \Big(\dfrac{da_0(\tau)}{d\tau} - 2\dfrac{du_0(\tau)}{d\tau}\Big)\xi^2{}_0(\tau)}{u_0(\tau)+a_0(\tau)} \qquad (2.10)$$

If we specify as follows:

$$q(\tau) = \xi^2{}_0(\tau) \qquad (2.11)$$

The second expression of (2.2), from equations (2.8) and (2.12):

$$\begin{cases} \dfrac{dq(\tau)}{d\tau} = 12 + 4m\sqrt{q(\tau)} - \dfrac{4sq(\tau)}{(1-\sqrt{q(\tau)})(u_0(\tau)+a_0(\tau))} - \dfrac{2sq(\tau)}{(u_0(\tau)+a_0(\tau))} \\ \dfrac{da_0(\tau)}{d\tau} = m(\dfrac{-2(u_0(\tau)+a_0(\tau))}{\sqrt{q(\tau)}} - s) \\ \dfrac{du_0(\tau)}{d\tau} = -\dfrac{s}{1-\sqrt{q(\tau)}} \end{cases} \quad (2.12)$$

The initial conditions for the above three unknown functions are as follows:

$$\begin{cases} q(\tau) = \xi_0^2(\tau), \quad q(0) = 0 \\ a_0(\tau) = 1 \\ u_0(\tau) = 0 \end{cases} \quad (2.13)$$

As a result, the problem given by the initial and boundary conditions of the equations (2.2) in dimensionless form (2.3)-(2.4) is brought to solve the Cauchy problem, that is, to find the solution of the system of equations (2.14) satisfying the conditions (2.15).
According to (2.15), q(0)=0, a $_0$ (0)=1, u $_0$ (0)=0 is taken, and at small values of *t* the functions look like this:

$$\begin{cases} \dfrac{dq(\tau)}{d\tau} = 12 \\ \dfrac{da_0(\tau)}{d\tau} = \dfrac{m}{\sqrt{3\tau}} - ms \\ \dfrac{du_0(\tau)}{d\tau} = -s \end{cases} \Rightarrow \begin{cases} q(\tau) = 12\tau \\ a_0(\tau) = 1 + \dfrac{2m\sqrt{\tau}}{\sqrt{3}} - ms\tau \\ u_0(\tau) = -s\tau \end{cases} \quad (2.14)$$

These functions are used in the first step of numerical solution of equation (2.15). In the next time steps, solutions are obtained using auxiliary programs based on the conditions (2.15) and (2.16).
**Determination of stresses in the stern using the "Maple" program.** The system of equations (2.14) was brought to the problem of finding a solution satisfying the conditions (2.15). Now let's consider solving this system of differential equations. It is known that the Saint-Venant parameter in the resulting system of equations $s = \dfrac{\sigma_0 l}{\mu u_0}$ describes the movement and determines the ratio of the masses of the two bodies involved in the impact process. $m = \dfrac{M_c}{M_T}$ since it can take different values, we pay attention to solving the problem numerically, that is, when creating a program, for different values of *s* and *m*.
Below in Figure 2 - for values the Saint-Venant parameter equal to 0.25, 0.5, 1 and 2, you can see the change of the adhesive-plastic boundary after impact for the case where the masses of the two objects are relatively equal (*m*=1.0).

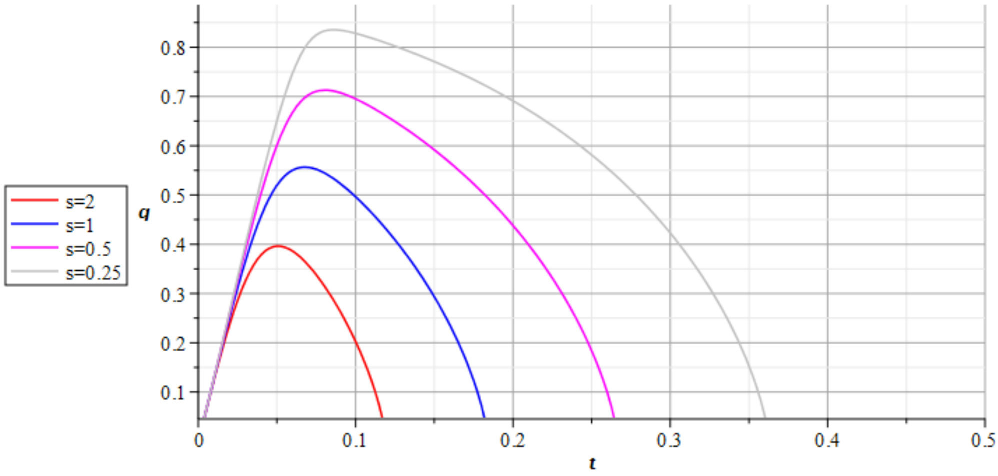


***Figure 2***

As can be seen from this graph, an expansion of the visco-plastic region can be seen in the early times after the start of the sturgeon hitting the solid wall. At some time t, the size (length) $x_0$ (t) of this sphere reaches a maximum, and then the size of the sphere decreases. As the value of the parameter *s* increases, i.e., without changing other parameters of the problem, the speed $V_0$ of the barge at impact or the decrease of the parameter m, which represents the viscosity property, leads to a decrease in the maximum value of the visco-plastic area, and vice versa, if the barge hits the wall at a high speed, its visco-plastic deformable part will be bigger.

It was also observed that the size of the viscous-plastic zone decreases when the length of the rod increases (while other parameters of the problem: the speed and the value of parameters m do not change). After the impact, when $x_0$ (t) reaches a maximum value at some time t = t*, the value of $x_0$ (t) decreases and finally becomes zero, that is, the viscous-plastic area disappears. This means that the velocities of the rigid area of the stern will be equal to zero, that is, the movement of the stern will stop completely.

It can be seen from Fig. 2 that in all cases examined, the second - hard area of the stern is always present, and this area does not deform even as a result of impact.

In Figure 3, m=1 when At different values of the parameter $\sigma$ *, the variation of the shear velocity* $u_0$ (t) in the non-deformable and visco-plastic limit of the stergenn over time t is given.

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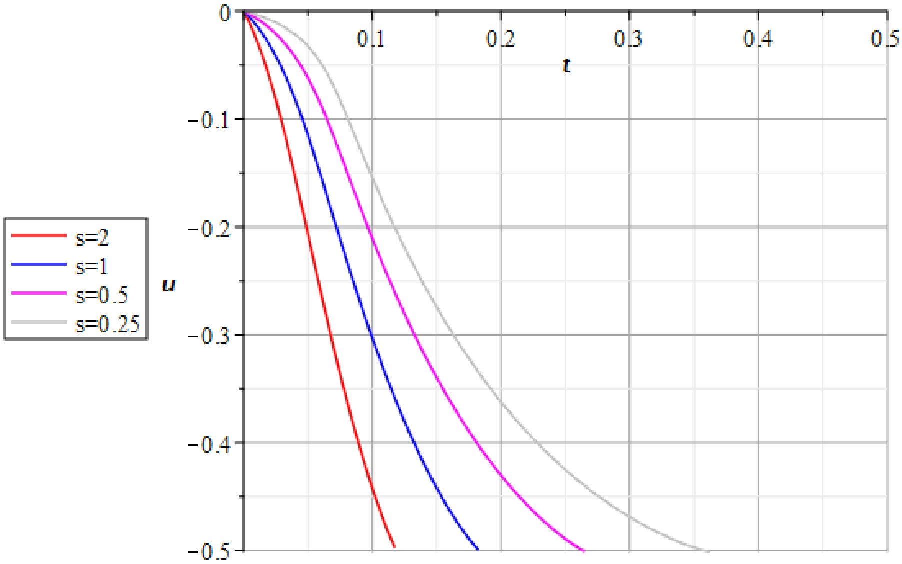


***Figure 3***

It can be seen from these pictures that the velocity of the viscous-plastic zone limit and the non-deformable part of the rod after hitting the solid wall decreases with time and becomes zero after a certain time. An increase in the parameter σ, i.e., a decrease in the shock velocity or an increase in the length of the boom $u_0(t)$ increases the intensity of the decay of the velocity with time and decreases the time to zero.

Now let's focus on the values of the tension that appears as a result of the shock in the sections of the stern:

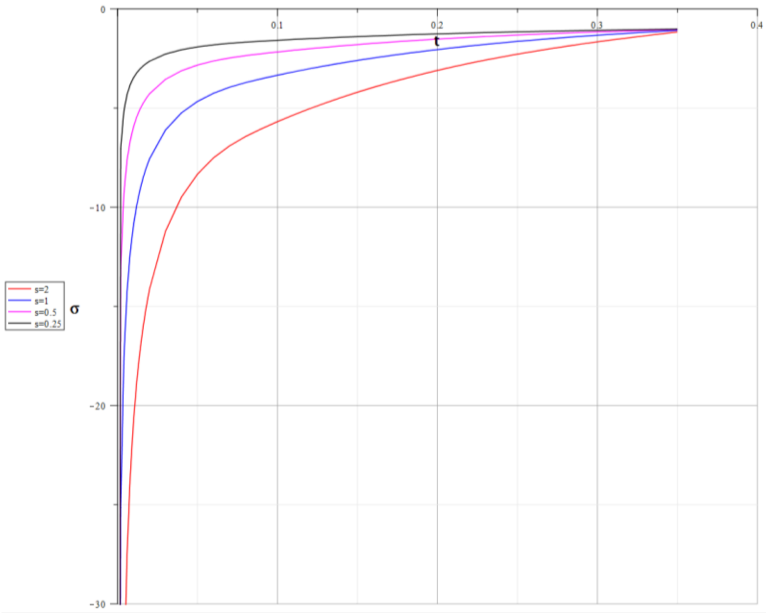


***Figure 4***

It can be seen from the above graph that as the value of the parameter σ increases, it can be observed that the stress approaches the limit stress after the impact occurs faster.

**SUMMARY**

1. The problem of the impact of an absolute solid body on a viscous plastic rod related to the mechanics of a deformable solid body is reduced to a diffusion problem.
2. When a solid object hits a visco-plastic rod, the size of its visco-plastic deformable area increases, reaches a maximum value, and then decreases to zero; it was observed that a certain part of the end of the stern, which was hit by a solid object, was always not deformed.
3. It is shown that the maximum size of the visco-plastic region decreases as the value of the Saint-Venant parameter increases.
4. The estimated values of the Saint-Venant parameter (0.25, 0.5, 1 and 2) tend to the limit value of stress after the end of the process.

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