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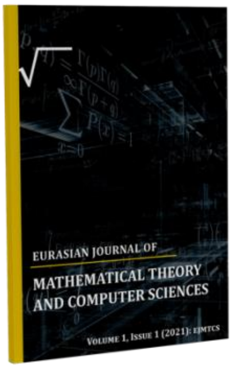

DIFFERENTIATION OF A FOUR-DIMENSIONAL NILPOTENT ALGEBRA

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ABSTRACT

Differentiation is one of the fundamental concepts of mathematics. Differentiations also play an important role in algebra. There are various generalizations of differentiations. These include antidifferentiations, δ-differentiations, ternary differentiations, and (α,β,γ)-differentiations. In this paper, the differentiation of four-dimensional nilpotent algebras is shown by proving local differentiation.

ДИФФЕРЕНЦИАЦИЯ ЧЕТЫРЕХМЕРНОЙ НИЛЬПОТЕНТНОЙ АЛГЕБРЫ

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Дифференциация, алгебра, локальное дифференцирование, линейное отображение.

ABSTRACT

Дифференциация — одно из фундаментальных понятий математики. Дифференциации также играют важную роль в алгебре. Существуют различные обобщения дифференциаций. К ним относятся антидифференциации, δ-дифференциации, тернарные дифференциации и (α,β,γ)-дифференциации. В данной работе дифференциация четырехмерных нильпотентных алгебр демонстрируется путем доказательства локальной дифференциации.

TO‘RT O‘LCHAMLI NILPOTENT ALGEBRANING DIFFERENSIALLASHI

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KEYWORDS

Differensiallash, Algebra, lokal differensiallash, chiziqli akslatirish.

ABSTRACT

Differensiallash matematikaning fundamental tushunchalaridan biri hisoblanadi. Differensiallashlar algebra fanida ham muhim o'rin tutadi. Differensiallashlarning turli umumlashmalari mavjud. Bular sarasiga antidifferensiallashlar, δ-differensiallashlar, ternar differensiallashlar va (α,β,γ)-differensiallashlar kiradi. Ushbu ishda to'rt o'lchamli nilpotent algebralarning differensiallashi isboti bilan ko'rsatilgan.

Kirish

Quyida biz to'rt o'lchamli nilpotent algebralarning differensiallashini keltiramiz.

R haqiqiy sonlar maydoni ustida aniqlangan $\{e_1, e_2, e_3, e_4\}$ bazisli nilpotent algebra N_2 berilgan. Bu algebrada ko'paytirish jadvali quyidagicha aniqlanadi:

$$e_1e_1 = e_2 \qquad e_2e_3 = e_4 \qquad e_3e_2 = e_4 \qquad e_3e_1 = e_4$$

N_2 algebrada ixtiyoriy x vektorni

$$x = x_1e_1 + x_2e_2 + x_3e_3 + x_1e_4$$

ko'rinishda yozish mumkin. $d : N_2 \to N_2$ akslantirishni qaraylik.

1-Ta'rif. Agar N algebaradan olingan ixtiyoriy x, y elementlar uchun

$$d(x \cdot y) = d(x) \cdot y + x \cdot d(y)$$

Tenglik bajarilsa d akslantirish qaralayotgan N algebrada differensiallash deb ataladi.

$d(x) = Ax$ chiziqli akslantirishni qaraylik, bu yerda A – 4×4 o'lchamli kvadrat matritsa. Ravshanki, agar N_2 algebaradan olingan ixtiyoriy x, y elementlar uchun

$$A(xy) = (Ax)y + x(Ay) \tag{1}$$

tenglik bajarilsa $d(x) = Ax$ akslantirish qaralayotgan N_2 algebrada differensiallashdan iborat bo'ladi.

1-Teorema. $d(x) = Ax$ akslantirish N_2 algebrada differensiallashdan iborat bo'lishi uchun A matritsa

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{3,3} & 0 \\ a_{4,1} & 0 & a_{4,3} & a_{3,3} \end{pmatrix} \tag{2}$$

ko'rinishda bo'lishi zarur va yetarli.

Isbot. Zarurligi. $d(x) = Ax$ akslantirish N_2 algebrada differensiallashdan iborat bo'sin. U holda

$$A=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}$$

Matritsa ixtiyoriy x, y vektorlar uchun (1) tenglikni qanoatlantirishi kerak.
Xususan $e_1e_1 = e_2$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_1)e_1 + e_1(Ae_1) = A(e_2)$$

qaraylik:

$$Ae_1\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}a_{1,1}\\a_{2,1}\\a_{3,1}\\a_{4,1}\end{pmatrix}=a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3+a_{4,1}e_4$$

$$Ae_2=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}\begin{pmatrix}0\\1\\0\\0\end{pmatrix}=\begin{pmatrix}a_{1,2}\\a_{2,2}\\a_{3,2}\\a_{4,2}\end{pmatrix}=a_{1,2}e_1+a_{2,2}e_2+a_{3,2}e_3+a_{4,2}e_4$$

$$Ae_3=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}=\begin{pmatrix}a_{1,3}\\a_{2,3}\\a_{3,3}\\a_{4,3}\end{pmatrix}=a_{1,3}e_1+a_{2,3}e_2+a_{3,3}e_3+a_{4,3}e_4$$

$$Ae_4=\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} & a_{1,4} \\ a_{2,1} & a_{2,2} & a_{2,3} & a_{2,4} \\ a_{3,1} & a_{3,2} & a_{3,3} & a_{3,4} \\ a_{4,1} & a_{4,2} & a_{4,3} & a_{4,4} \end{pmatrix}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}=\begin{pmatrix}a_{1,4}\\a_{2,4}\\a_{3,4}\\a_{4,4}\end{pmatrix}=a_{1,4}e_1+a_{2,4}e_2+a_{3,4}e_3+a_{4,4}e_4$$

$$(Ae_1)e_1=\left(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3+a_{4,1}e_4\right)e_1=a_{1,1}e_2+a_{3,1}e_4$$

$e_1(Ae_1)$= $e_1\left(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3+a_{4,1}e_4\right)=a_{1,1}e_2$

$$a_{1,1}e_2+a_{3,1}e_4+a_{1,1}e_2=a_{1,2}e_1+a_{2,2}e_2+a_{3,2}e_3+a_{4,2}e_4$$

bundan

$$a_{2,2}=2a_{1,1},\qquad a_{1,2}=0,\qquad a_{3,2}=0\quad a_{4,2}=a_{3,1}\quad Ae_2=2a_{1,1}e_2+a_{3,1}e_4$$

tengliklarni aniqlaymiz.
Endi $e_1e_2 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_1)e_2+e_1(Ae_2)=A(0)$$

qaraylik:

$$(Ae_1)e_2=(a_{1,1}e_1+a_{2,1}e_2+a_{3,1}e_3+a_{4,1}e_4)e_2=a_{3,1}e_4$$

$$e_1(Ae_2)=\ e_1\left(2a_{1,1}e_2+a_{3,1}e_4\right)=0$$

$$a_{3,1}e_4=0$$

bundan

$$a_{3,1}=0,\qquad a_{4,2}=0\quad Ae_2=2a_{1,1}e_2\quad Ae_1=a_{1,1}e_1+a_{2,1}e_2+a_{4,1}e_4$$

tengliklarni aniqlaymiz.
Endi $e_1e_3 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_1)e_3+e_1(Ae_3)=A(0)$$

qaraylik:

$$(Ae_1)e_3=\left(a_{1,1}e_1+a_{2,1}e_2+a_{4,1}e_4\right)e_3=a_{2,1}e_4$$

$$e_1(Ae_3)=e_1\left(a_{1,3}e_1+a_{2,3}e_2+a_{3,3}e_3+a_{4,3}e_4\right)=a_{1,3}e_2$$

$$a_{2,1}e_4+a_{1,3}e_2=0$$

bundan

$$a_{1,3}=0,\quad a_{2,1}=0\quad Ae_1=a_{1,1}e_1+a_{4,1}e_4\quad Ae_3=a_{2,3}e_2+a_{3,3}e_3+a_{4,3}e_4$$

tengliklarni aniqlaymiz.

Endi $e_1e_4=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_1)e_4+e_1(Ae_4)=A(0)$$

qaraylik:

$$(Ae_1)e_4=(a_{1,1}e_1+a_{4,1}e_4)e_4=0$$

$$e_1(Ae_4)=e_1\big(a_{1,4}e_1+a_{2,4}e_2+a_{3,4}e_3+a_{4,4}e_4\big)=a_{1,4}e_2$$

$$a_{1,4}e_2=0$$

bundan

$$a_{1,4}=0,\qquad Ae_4=a_{2,4}e_2+a_{3,4}e_3+a_{4,4}e_4$$

tengliklarni aniqlaymiz.

Endi $e_2e_1=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_2)e_1+e_2(Ae_1)=A(0)$$

qaraylik:

$$(Ae_2)e_1=(2a_{1,1}e_2)e_1=0$$

$$e_2(Ae_1)=e_2(a_{1,1}e_1+a_{4,1}e_4)=0$$

$$0=0$$

bo'ldi.

Endi $e_2e_2=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_2)e_2+e_2(Ae_2)=A(0)$$

qaraylik:

$$(Ae_2)e_2=(2a_{1,1}e_2)e_2=0$$

$$e_2(Ae_2)=e_2(2a_{1,1}e_2)=0$$

$$0=0$$

bo'ldi.

Endi $e_2e_3=e_4$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_2)e_3+e_2(Ae_3)=A(e_4)$$

qaraylik:

$$(Ae_2)e_3=(2a_{1,1}e_2)e_3=2a_{1,1}e_4$$

$$e_2(Ae_3)=e_2\big(a_{2,3}e_2+a_{3,3}e_3+a_{4,3}e_4\big)=a_{3,3}e_4$$

$$2a_{1,1}e_4+a_{3,3}e_4=a_{2,4}e_2+a_{3,4}e_3+a_{4,4}e_4$$

bundan

$$a_{2,4}=0,\quad a_{3,4}=0,\quad a_{4,4}=2a_{1,1}+a_{3,3}\quad Ae_4=(2a_{1,1}+a_{3,3})e_4$$

tengliklarni aniqlaymiz.

Endi $e_2e_4=0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_2)e_4+e_2(Ae_4)=A(0)$$

qaraylik:

$$(Ae_2)e_4=(2a_{1,1}e_2)e_4=0$$

$$e_2(Ae_4)=e_2(2a_{1,1}+a_{3,3})e_4=0$$

$$0=0$$

bo'ldi.

Endi $e_3e_1 = e_4$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_3)e_1 + e_3(Ae_1) = A(e_4)$$

qaraylik:

$$(Ae_3)e_1 = \left(a_{2,3}e_2 + a_{3,3}e_3 + a_{4,3}e_4\right)e_1 = a_{3,3}e_4$$
$$e_3(Ae_1) = e_3(a_{1,1}e_1 + a_{4,1}e_4) = a_{1,1}e_4$$
$$a_{3,3}e_4 + a_{1,1}e_4 = (2a_{1,1} + a_{3,3})e_4$$
$$a_{3,3} + a_{1,1} = 2a_{1,1} + a_{3,3}$$

bundan

$$a_{1,1} = 0, \quad Ae_1 = a_{4,1}e_4 \quad Ae_2 = 0 \quad Ae_4 = a_{3,3}e_4$$

tengliklarni aniqlaymiz.

Endi $e_3e_2 = e_4$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_3)e_2 + e_3(Ae_2) = A(e_4)$$

qaraylik:

$$(Ae_3)e_2 = (a_{2,3}e_2 + a_{3,3}e_3 + a_{4,3}e_4)e_2 = a_{3,3}e_4,$$
$$e_3(Ae_2) = e_3(0) = 0$$
$$a_{3,3}e_4 = a_{3,3}e_4$$
$$0 = 0$$

bo‘ldi.

Endi $e_3e_3 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_3)e_3 + e_3(Ae_3) = A(0)$$

qaraylik:

$$(Ae_3)e_2 = \left(a_{2,3}e_2 + a_{3,3}e_3 + a_{4,3}e_4\right)e_3 = a_{2,3}e_4$$
$$e_3(Ae_2) = e_3\left(a_{2,3}e_2 + a_{3,3}e_3 + a_{4,3}e_4\right) = a_{2,3}e_4$$
$$a_{2,3} + a_{2,3} = 0$$

bundan

$$a_{2,3} = 0 \qquad Ae_3 = a_{3,3}e_3 + a_{4,3}e_4$$

tengliklarni aniqlaymiz

Endi $e_3e_4 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_3)e_4 + e_3(Ae_4) = A(0)$$

qaraylik:

$$(Ae_3)e_4 = \left(a_{3,3}e_3 + a_{4,3}e_4\right)e_4 = 0,$$
$$e_3(Ae_4) = e_3(a_{3,3}e_4) = 0$$
$$0 = 0$$

bo‘ldi.

Endi $e_4e_1 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_4)e_1 + e_4(Ae_1) = A(0)$$

qaraylik:

$$(Ae_4)e_1 = (a_{3,3}e_4)e_1 = 0,$$
$$e_4(Ae_1) = e_4(a_{4,1}e_4) = 0$$
$$0 = 0$$

bo‘ldi.

Endi $e_4e_2 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_4)e_2 + e_4(Ae_2) = A(0)$$

qaraylik:

$$(Ae_4)e_2 = (a_{3,3}e_4)e_2 = 0,\ \ e_4(Ae_2) = e_4(0) = 0$$
$$0 = 0$$

bo'ldi.

Endi $e_4e_3 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_4)e_3 + e_4(Ae_3) = A(0)$$

qaraylik:

$$(Ae_4) = (a_{3,3}e_4)e_3 = 0,\ \ e_4(Ae_3) = e_4(a_{3,3}e_3 + a_{4,3}e_4) = 0$$
$$0 = 0$$

bo'ldi.

Endi $e_4e_4 = 0$ tenglikdan foydalanib quyidagi ayniyatni

$$(Ae_4)e_4 + e_4(Ae_4) = A(0)$$

qaraylik:

$$(Ae_4)e_4 = (a_{3,3}e_4)e_4 = 0,\ \ e_4(Ae_4) = e_4(a_{3,3}e_4) = 0$$
$$0 = 0$$

bo'ldi.

Yetarliligi. Endi A matritsa (2) ko'rinishda bo'lsa $d(x) = Ax$ chiziqli akslantirish differensiallashdan iboratligini ko'rsataylik.

$$xy = (x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4)(y_1e_1 + y_2e_2 + y_3e_3 + y_4e_4) =$$
$$= x_1y_1e_2 + (x_2y_3 + x_3y_1 + x_3y_2)e_4$$

Tenglikka ko'ra

$$A(xy) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{3,3} & 0 \\ a_{4,1} & 0 & a_{4,3} & a_{3,3} \end{pmatrix} \begin{pmatrix} 0 \\ x_1y_1 \\ 0 \\ x_2y_3 + x_3y_1 + x_3y_2 \end{pmatrix} =$$
$$= a_{3,3}x_2y_3 + a_{3,3}x_3y_1 + a_{3,3}x_3y_2$$

boshqa tamondan

$$Ax = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{3,3} & 0 \\ a_{4,1} & 0 & a_{4,3} & a_{3,3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} =$$
$$= a_{3,3}x_3e_3 + (a_{41}x_1 + a_{4,3}x_3 + a_{3,3}x_4)e_4$$
$$(Ax)y = (a_{3,3}x_3e_3 + (a_{41}x_1 + a_{4,3}x_3 + a_{3,3}x_4)e_4)(y_1e_1 + y_2e_2 + y_3e_3 + y_4e_4) =$$
$$= a_{3,3}x_3y_1e_4 + a_{3,3}x_3y_2e_4$$
$$Ay = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{3,3} & 0 \\ a_{4,1} & 0 & a_{4,3} & a_{3,3} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} =$$
$$= a_{3,3}y_3e_3 + (a_{41}y_1 + a_{4,3}y_3 + a_{3,3}y_4)e_4$$
$$x(Ay) = (x_1e_1 + x_2e_2 + x_3e_3 + x_4e_4)(a_{3,3}y_3e_3 + (a_{41}y_1 + a_{4,3}y_3 + a_{3,3}y_4)e_4) =$$
$$= a_{3,3}x_2y_3e_4$$

Yuqoridagi xisoblashlar (2) tenglik to'g'riligini ko'rsatadi. **Teorema isbotlandi.**

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