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POLINOMIAL XALQALAR IDEALLARINING GRYOBNER BAZISLARINI TOPISH HAQIDA

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ABSTRACT

Ideallarning Gryobner bazislarining haqiqiy ahamiyati shundan iboratki, ularni hisoblash mumkin. Bruno Buxberger o'zining ustozi Volfgang Gryobnerning ilmiy ishlarining ta'sirida 1965 yil fanga Gryobner bazislari tushunchasini kiritdi. Shunday bazislarni hisoblashning Buxbereger algoritmi polinomial xalqalar nazariyasiga qo'shilgan muhim hissa bo'ldi. Biz bu ishda ushbu algoritmni to'liq bermaymiz. Ushbu ishda idealning bazisi qanday shartlarda Gryobner bazisi tashkil qilishi haqidagi tasdiqlar va misol keltiriladi. Algoritmni to'liq o'rganish uchun Кокс, Литтл va O'шуларнинг monografiyasiga [2] murojaat qilish mumkin.

1-Ta'rif. $f, g \in K[x_1, \cdots, x_n]$ nol bo'lmagan polinomlar bo'lsin.

1) $multideg(f) = \alpha, multideg(g) = \beta$ va $\gamma_i = \max\{\alpha_i, \beta_i\}$, $i = 1, 2, \cdots, n$ bo'lsin. $\gamma = (\gamma_1, \cdots, \gamma_n)$ bo'lsin. U holda x^γ monom $LM(f)$ va $LM(g)$ larning eng kichik umumiy karralisi deyiladi va

$$L = x^\gamma = LCM(LM(f), LM(g))$$

shaklda yoziladi.

2) $S(f,g) = \frac{x^\gamma}{LT(f)} \cdot f - \frac{x^\gamma}{LT(g)} \cdot g$ polinom f va g polinomlarning S-polinomi deb ataladi.

1-misol. $y > x$ bo'lsin**.** U holda $R[x, y]$ da $grlex$-tartiblash bilan $f_1 = y^2 - yx^2$ va $f_2 = y^3x - y^2 + y$

$$multideg(f_1) = (2{,}0), \qquad multideg(f_2) = (3{,}1)$$

bo'ladi. Shunday qilib, $L = y^3x$ va

$S(f_1, f_2) = \frac{y^3x}{y^2} \cdot f_1 - \frac{y^3x}{y^3x} \cdot f_2 = yx \cdot f_1 - 1 \cdot f_2 = y^3x - y^2x^3 - y^3x + y^2 - y = -y^2x^3y^2 - y.$

S-polinom $S(f,g)$ ni kiritishdan maqsad polinomlarning bosh hadini nolga aylantirishdan iborat. Quyidagi lemma ana shunday jarayonning barchasida S -polinomning borligini ko'rsatadi.

1-Lemma. $\sum_{i=1}^{t} c_i x^{\alpha(i)} g_i$ *yig'indini qaraymiz,* $c_1, \cdots, c_t$ $(c_i \neq 0)$ *lar konstantalar va* $\alpha(i) + multideg(g_i) = \delta \in W^n$.

Agar $multideg\left(\sum_{i=1}^{t} c_i x^{\alpha(i)} g_i\right) < \delta$ *bo'lsa, u holda shunday* c_{jk} *konstantalar mavjudki,*

$\sum_{i=1}^{t} c_i x^{\alpha(i)} g_i = \sum_{j,k} c_{jk} x^{\delta-\gamma_{jk}} S(g_j, g_k),$ (1)

bo'ladi, bu yerda $x^{\gamma_{jk}} = LCM(LM(g_j), LM(g_k))$. *Bundan tashqari har bir* $x^{\delta-\gamma_{jk}} S(g_j, g_k)$ *ning umumiy darajasi* $multidegree < \delta$ *bo'ladi.*

(1) tenglikning chap tomonidagi yig'indining har bir qo'shiluvchisi $c_i x^{\alpha(i)} g_i$ ning umumiy darajasi $multidegree\ \delta$ bo'ladi. Shuning uchun bu qo'shiluvchilar ixchamlashtirilgandan so'ng bosh hadlar yo'qoladi. (1) ning o'ng tomonidagi yig'indining har bir qo'shiluvchisi $c_{jk} x^{\delta-\gamma_{jk}} S(g_j, g_k)$ ninng umumiy darajasi esa $multidegree < \delta$ bo'ladi. Demak, bu yig'indida ixchamlashtirishlar amalga oshirilgan bo'ladi. Bu yerdan ko'rinib turibdiki, S-polinomlar ixchamlashtirishlarni amalga oshirishda yordam berar ekan.

1-Teorema. *$I -\ K[x_1, \cdots, x_n]$ xalqaning nol bo'lmagan ideali bo'lsin. U holda I idealning biror $G = \{g_1, \dots, g_t\}$ bazisi $S(g_i, g_j)$ G ga (biror tartiblash bo'yicha) bo'linganda hosil bo'lgan qoldiq $\overline{S(g_i, g_j)}^G$ barcha i, j $(i \neq j)$ lar uchun nol bo'lganda va faqat shu holdagina I idealning Gryobner bazisidan iborat bo'ladi.*

2-tarif. I $K[x_1, .., x_n]$ xalqaning ideali, G esa shu idealning Gryobner bazisi bo'lsin. Agar quyidagi

1) barcha $g \in G$ lar uchun $LC(g) = 1$ bo'lsa

2) barcha $g \in G$ lar uchun $LT(g) \notin < LT(G/\{g\}) >$

shartlar bajarilsa G ga I idealning minimal Gryobner bazisi deyiladi.

3-tarif. I $K[x_1, .., x_n]$ xalqaning ideali, G esa shu idealning Gryobner bazisi bo'lsin. Agar

1)barcha $g \in G$ lar uchun $LC(g) = 1$ bo'lsa

2) barcha $g \in G$ lar uchun g ning birorta xam monomiali $< LT(G/\{g\}) >$ ga tegishli bo'lmasa G ga I idealning keltirilgan Gryobner bazisi deyiladi

2-teorema I $K[x_1, \dots, x_n]$ xalqaning ideali bo'lsin. U holda yagona monomial tartiblash bo'yicha I ideal yagona keltirilgan Gryobner bazisiga ega bo'ladi.

2-misol. Quidagi polinomial tenglamalar sistemasini qaraymiz

$$\begin{cases} x^2 - y = 0 \\ x^2 z = 0 \end{cases}$$

$R[x, y, z]$ da $I = < x^2 - y, x^2 z >$ idealning keltirilgan Gryobner bazisini toping.

Yechish: Dastlab biz $\{f_1, f_2\}$ sistema Gryobner bazisi bo'lish yoki bo'lmasligini tekshiramiz. Buning uchun $S(f_1, f_2)$ ni topamiz:

$multdeg(f_1) = (2,0,0), LT(f_1) = x^2, LM(f_1) = x^2, LC(f_1) = 1$

$multdeg(f_2) = (2,0,1), LT(f_2) = x^2 z, LM(f_2) = x^2 z, LC(f_2) = 1$

$S(f_1, f_2) = \frac{x^2 z}{x^2}(x^2 - y) - \frac{x^2 z}{x^2 z}(x^2 z) = -yz$ bo'ladi.

Endi $S(f_1, f_2) = -yz$ ni f_1, f_2 larga qoldiqli bo'lamiz

$$\begin{array}{c|c} & a_1 = 0 \\ & a_2 = 0 \\ \hline x^2 - y & -yz \\ x^2 z & 0 \\ \hline & -yz \end{array}$$

Ko'rinib turibdiki, qoldiq $-yz \neq 0$. Shuning uchun bu qoldiqni idealni tashkil etuvchi polinomlar to'plamiga kiritamiz. $F = (f_1, f_2, f_3)$ bo'lsin, bu yerda $f_3 = -yz$. U holda $S(f_1, f_2) = -yz$ bo'ladi natijada $\overline{S(f_1, f_2)}^F = 0$ bo'ladi. Endi f_1 va f_3 ni qaraymiz.Bu holda
$multdeg(f_3) = (0,1,1)$, $LT(f_3) = -yz$, $LM(f_3) = yz$, $LC(f_3) = -1$

$S(f_1, f_3) = \frac{x^2yz}{x^2}(x^2 - y) - \frac{x^2yz}{-yz}(-yz) = -y^2z$ bo'ladi.

Endi $S(f_1, f_3) = -y^2z$ ni f_1, f_2, f_3 larga qoldiqli bo'lamiz

$$
\begin{array}{c|l}
 & a_1 = 0 \\
 & a_2 = 0 \\
 & a_3 = y \\
\hline
x^2 - y & -y^2z \\
x^2z & -y^2z \\
-yz & \\
\hline
 & \quad 0
\end{array}
$$

Demak $\overline{S(f_1, f_3)}^F = 0$ bo'ladi.

Endi f_2 va f_3 larni ko'rib chiqamiz. Bu holda
$S(f_1, f_3) = \frac{x^2yz}{x^2z}(x^2z) - \frac{x^2yz}{-yz}(-yz) = 0$, $\overline{S(f_2, f_3)}^F = 0$ bo'ladi .Shunday qilib, $F = (f_1, f_2, f_3)$ sistemani xosil qildik va barcha $1 \leq i < j \leq 3$ lar uchun $\overline{S(f_i, f_j)}^F = 0$ ekanligini ko'rsatdik. Demak $I = < x^2 - y, x^2z >$ idealning Gryobner bazisi $F = \{x^2 - y, x^2z, -yz\}$ dan iborat ekan.

Keltirilgan Gryobner bazisini topamiz. Buning uchun

$$
\begin{cases}
x^2 - y = 0 \\
x^2z = 0 \\
-yz = 0
\end{cases}
$$

tashkil etuvchilarining bosh koffisiyentlarini 1 ga keltiramiz, ya'ni f_3 polinomni -1 ga ko'paytiramiz. Endi $LT(f_2) = x^2z = zLT(f_1)$ bo'lganligi uchun f_2 polinomni bazis palinomlari safidan chiqarishimiz mumkin. Qolgan yasovchilardan birortasining ham bosh koeffisiyenti bir-birining bosh koeffisiyentiga bo'linmaydi. Demak $x^2 - y$, yz polinomlar I idealining minimal Gryobner bazisini tashkil qiladi, shu bilan bir qatorda $x^2 - y, yz$ polinomlar I idealning keltirilgan Gryobner bazisini tashkil qiladi, chunki $x^2 \notin < yz >$,$yz \notin < x^2 >$,$y \notin < yz >$ shartlar o'rinli.

Idealning Gryobner bazisini topishda Maple dasturidan foydalanish mumkin. Ko'rib chiqqan misolimizni Maple dasturi yordamida yechish quyida keltirilgan.

```
> with(Groebner):
> F := [x^2 - y, x^2*z];
F := [x^2 - y, x^2 z]
> Basis(F, plex(x, y, z)); # lexicographic order with x > y>z
[yz, x^2 - y]
> Basis(F, tdeg(x, y, z)); # graded  reverse  lexicographic order
[yz, x^2 - y]
```

```
> Basis(F,'tord');    # choose a term order and assign it to tord
```
$[y z, x^2 - y]$

```
> tord;
```
$tdeg(x, y, z)$

> Basis(F, 'tord', order='grlex'); # choose a graded lex order

$[x^3 - 3xy, x^2 y - 2y^2 + x, xy^2 + x^2, y^3 + xy]$

> tord;

$grlex(y, x)$

> Basis(F, plex(x,y), characteristic=3); # computation over Z[3]

$[y, x^2]$

> G, C := Basis(F, plex(x,y), output=extended); # compute a transformation matrix

$G, C := [y, x^2], \left[\left[-1, \frac{1}{z}\right], \left[0, \frac{1}{z}\right]\right]$

> [seq(expand(add(C[i][j]*F[j], j=1...nops(F))), i=1..nops(C))];

$[y, x^2]$

We construct a PolynomialIdeal data structure, which automatically keeps track of known Groebner bases.

> with(PolynomialIdeals):

> J := <F>;

$J := \langle x^2 z, x^2 - y \rangle$

> IdealInfo[KnownGroebnerBases](J);

$\{plex(x, y), plex(x, y, z), tdeg(x, y, z), tdeg(y, z, x)\}$

The commands below do not perform any Groebner basis computations. The known Groebner bases are examined to see if a computation can be avoided.

> HilbertDimension(J);

1

> UnivariatePolynomial(y, J);

0

> Homogenize(J, h);

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