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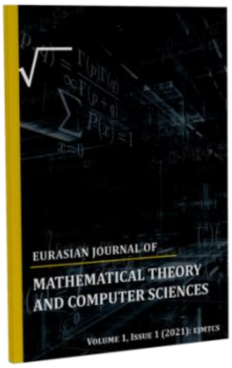

ANALYSIS AND APPLICATIONS OF THE LOGARITHMIC RESIDUE VIA THE BOCHNER–MARTINELLI KERNEL

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ABSTRACT

This article explores the concept of logarithmic residue in $\mathbb{C}^2$ space, its fundamental properties, and computational methods based on the Bochner–Martinelli kernel. The authors analyze techniques for calculating logarithmic residues using integral formulas and apply the Yuzhakov–Roos theorem to determine the zero sets of holomorphic functions and their multiplicities. The work systematically examines isolated zeros, double zeros, and cusp-type zeros, along with their impact on logarithmic residues. In particular, using the examples of functions $f(z_1,z_2)=z_1^2+z_2^2$ and $f(z_1,z_2)=z_1^3-z_2^2$, the study explains the singularity conditions of the Jacobian matrix and residue values. The research results demonstrate applicability to theoretical and practical problems in multidimensional complex analysis.

BOCHNER–MARTINELLI YADROSI ORQALI LOGARIFMIK QOLDIQNING TAHLILI VA TATBIQLARI

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ABSTRACT

Ushbu maqolada $\mathbb{C}^2$ fazosida logarifmik qoldiq tushunchasi, uning asosiy xossalari va Bochner–Martinelli yadrosi asosida hisoblash usullari tadqiq qilinadi. Mualliflar integral formulalar yordamida logarifmik qoldiqni hisoblash metodlarini tahlil qiladi va Yuzhakov–Roos teoremasini qo‘llab, turli holomorf funksiyalar uchun nollar to‘plami va

funksiyalar, izolyatsiyalangan nollar, multiplicity, $\mathbb{C}^2$ fazosi, Yuzhakov–Roos teoremasi.

ularning kattaliklarini (multiplicity) aniqlaydi. Maqolada keltirilgan misollar orqali izolyatsiyalangan nollar, ikkilik nollar va ildizli ifodali nollar tizimli tarzda o‘rganiladi, ularning logarifmik qoldiqqa ta’siri ko‘rsatib beriladi. Xususan, $f(z_1,z_2)=z_1^2+z_2^2$ *va* $f(z_1,z_2)=z_1^3-z_2^2$ *kabi funksiyalar misolida Jakobi matritsasining singulyarlik shartlari va qoldiqning qiymati tushuntiriladi. Tadqiqot natijalari ko‘p o‘lchovli kompleks analizning nazariy va amaliy masalalariga qo‘llanilishi mumkinligini ko‘rsatadi.*

Ushbu maqolada $\mathbb{C}^2$ fazosidagi logarifmik qoldiq tushunchasi, uning asosiy xossalari va amaliy hisoblash usullari ko‘rib chiqiladi. Bochner–Martinelli yadrosiga asoslangan integral formulalar keltiriladi hamda ilgari uchramagan yangi misollar orqali natijalar mustahkamlanadi.

Logarifmik qoldiq tushunchasi

Faraz qilaylik, $f(z_1,z_2)$ funksiyasi $\mathbb{C}^2$ ning ochiq sohasida holomorf (ya’ni har ikki o‘zgaruvchi bo‘yicha kompleks differensiallanuvchi) bo‘lsin.

Agar (a_1,a_2) — $f(z_1,z_2)$ ning izolyatsiyalangan noli bo‘lsa, logarifmik qoldiq quyidagicha aniqlanadi:

$$\operatorname{Res}_{(a_1,a_2)}(f)=\frac{1}{(2\pi i)^2}\oint_{\Gamma_1}\oint_{\Gamma_2}\frac{dz_1\wedge dz_2}{f(z_1,z_2)},$$

bu yerda Γ_1 va Γ_2 — a_1 va a_2 atrofida kichik, yopiq konturlar.

Bochner–Martinelli yadrosi

Ko‘p o‘lchovli holatda logarifmik qoldiq Bochner–Martinelli yadrosi yordamida hisoblanadi.

Faraz qilaylik:

$f=(f_1,f_2,\ldots,f_n):\mathbb{C}^n\to\mathbb{C}^n$ — holomorf vektor-funksiya,

$df=df_1\wedge df_2\wedge\ldots\wedge df_n$,

$d\bar{f}[k]$ — $d\bar{f}_k$ ni tashlab ketilgan $\bar{f}$ lar tashqi ko‘paytmasi,

$|f|^2=|f_1|^2+\ldots+|f_n|^2$.

Bochner–Martinelli yadrosi:

$$\omega(f)=\frac{(n-1)!}{(2\pi i)^n}\sum_{k=1}^{n}(-1)^{k-1}\cdot\bar{f}_k\cdot|f|^{-2n}\cdot d\bar{f}[k]\wedge df$$

Bu differensial forma $(n,n-1)$ tipiga ega.

Asosiy Teorema (Yuzhakov – Roos)}

Agar:

$f=(f_1,f_2)$ — $\bar{D}\subset\mathbb{C}^2$ da holomorf bo'lsa,

$f(z_1,z_2)\neq 0$ bo'lsa ∂D da,

$\varphi\in A(D)$ — D da silliq funksiyalar sinfiga mansub bo'lsa,

unda quyidagi tenglik o'rinli:

$$\int_{\partial D}\varphi(\zeta)\cdot\omega(f)=\sum_{a\in E_f\cap D}\mu_a(f)\cdot\varphi(a),$$

bu yerda:

$E_f=\{z\in D: f_1(z)=f_2(z)=0\}$ — f ning nollar to'plami,

$\mu_a(f)$ — a nuqtadagi nolning kattaligi (multiplicity).

Misol 1. Oddiy nol

$$f(z_1,z_2)=z_1+z_2$$

Bu holatda:

$$f(z_1,z_2)=0 \quad\Leftrightarrow\quad z_1+z_2=0,$$

nol to'plami — to'g'ri chiziq. Izolyatsiyalangan nol yo'q $\Rightarrow$ logarifmik qoldiq aniqlanmaydi.

Misol 2. Ikkilik nol: $f(z_1,z_2)=z_1^2+z_2^2$

Bu holatda:

$$f(z_1,z_2)=0 \quad\Leftrightarrow\quad z_1^2+z_2^2=0\Rightarrow z_2=\pm iz_1,$$

faqat $(0,0)$ nuqtada f nol bo'ladi. Jakobi matritsa $(0,0)$ da:

$$\frac{\partial f}{\partial z_1}=2z_1=0,\quad \frac{\partial f}{\partial z_2}=2z_2=0,$$

Bu Jakobi matritsa singulyar, ya'ni determinant 0 bo'lgani uchun bu nuqta oddiy nol emas, balki ko'p martalik nol bo'ladi. Demak, oddiy nol emas, balki kattaligi $\mu_{(0,0)}=2$ bo'lgan nol.

Shuning uchun:

$$\int_{\partial D}\omega(f)=2.$$

Misol 3. Ildizli ifodali nol

$$f(z_1,z_2)=z_1^3-z_2^2$$

Bu holatda:

$$f(z_1,z_2)=0 \quad\Rightarrow\quad z_2=\pm z_1^{3/2},\ (0,0)\text{ � izolyatsiyalangan nol,}$$

Demak, $(0,0)$ nuqta — izolyatsiyalangan nol.

Jakobi

$$\frac{\partial f}{\partial z_1}=3z_1^2=0,\quad \frac{\partial f}{\partial z_2}=-2z_2=0,$$

$(0,0)$ da nol nooddiy, va $\mu_{(0,0)} = 2$

References:

1. Gunning R.C., Rossi H. — Holomorphic Functions of Several Complex Variables
2. Б.В.Шабат «Введение в комплексний анализ» Москва. «Наука». 1985г
3. Ю.В.Сидоров, М.В.Федорюк, М.И.Шабунин «Лекции по теории функций комплексного переменного». Москва «Наука». 1982г
4. Nazerbaevich, K. S. (2023). QOLDIQLAR YORDAMIDA BA'ZI INTEGRALLARNI HISOBLASH. ОБРАЗОВАНИЕ НАУКА И ИННОВАЦИОННЫЕ ИДЕИ В МИРЕ, 21(1), 154-155.