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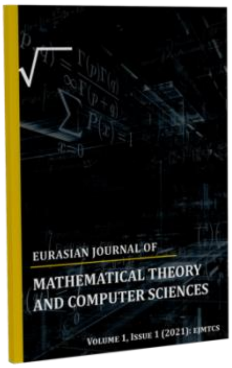

CALCULATING THE DISTANCE BETWEEN TWO POINTS IN SPACE: VECTOR GEOMETRY AND MATHEMATICAL ANALYSIS APPROACH

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ABSTRACT

This article studies one of the important issues in analytical geometry - methods for determining the shortest distance from a point to a straight line. From a geometric point of view, this distance is considered as the length of the perpendicular drawn from the point to the straight line. The article discusses in detail the basic formula used to calculate this distance, its mathematical foundations and the rules for its use. During the study, the formula for calculating the distance given the general equation of a straight line (Ax+By+C=0) and an arbitrary point (x_0, y_0) *. is analyzed. This topic is of great importance not only theoretically, but also in practical areas. In particular, it is widely used in engineering to determine the optimal distance between structures, and in the fields of computer graphics and physics to calculate spatial relationships between objects. The article can be a useful guide in mastering this formula and applying it to real problems.*

ВЫЧИСЛЕНИЕ РАССТОЯНИЯ МЕЖДУ ДВУМЯ ТОЧКАМИ В ПРОСТРАНСТВЕ: ВЕКТОРНАЯ ГЕОМЕТРИЯ И МАТЕМАТИЧЕСКИЙ АНАЛИЗА

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ABSTRACT

В данной статье рассматривается один из важных вопросов аналитической геометрии — методы

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KEYWORDS

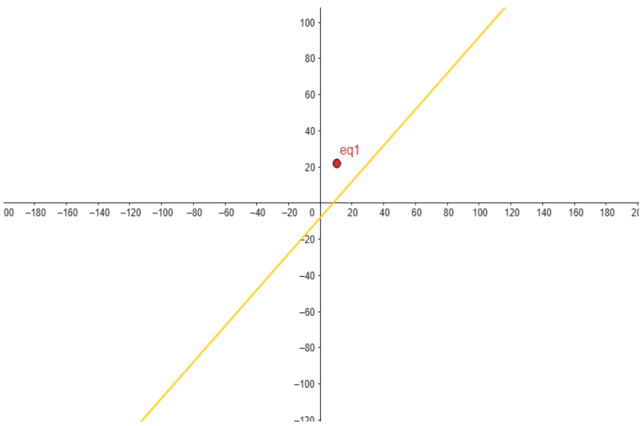
Векторы, расстояние между двумя точками, формула расстояния, интеграл, кривая, параметрическое уравнение, производная, точка, декартова система координат.

определения кратчайшего расстояния от точки до прямой. С геометрической точки зрения это расстояние рассматривается как длина перпендикуляра, проведенного из точки к прямой. В статье подробно рассматривается основная формула, используемая для вычисления этого расстояния, ее математические основы и правила ее использования. В ходе исследования анализируется формула для вычисления расстояния по общему уравнению прямой (Ax+By+C=0) и произвольной точке (x_0,y_0). Данная тема имеет большое значение не только в теоретическом, но и в практическом плане. В частности, она широко применяется в инженерии для определения оптимального расстояния между конструкциями, а в областях компьютерной графики и физики — для расчета пространственных отношений между объектами. Статья может стать полезным руководством при освоении этой формулы и применении ее к реальным задачам.

Calculating the distance between two points is one of the most important concepts in geometry. There are several ways to determine this distance. The most commonly used of them are finding distances based on vectors and determining the length along a curve by integral analysis.

In this article, in addition to both methods, another method is justified, illustrated with formulas, graphical approaches, and real-life applications.

Determining the distance from a point to a straight line is one of the most important problems in analytical geometry. This distance is always determined by the perpendicular drawn from a given point to the line.



Let $(x_0; y_0)$ be the point of intersection of a perpendicular line drawn from the point, lying on the given point Q(x;y). Then the distance is found as follows:

$$AD = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

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$(x_0; y_0)$ — is the given point, Q(x;y)-let it be the point of intersection of the perpendicular line drawn from the point lying on the straight line. Then the distance is found as follows:

$$AD = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

Proof:

We are given the equation of a straight line: Ax+By+C=0 and the point is F$(x_0; y_0)$ known. To find the distance from this straight line to the point:

1. We draw a perpendicular from the point to the line
2. We find the point of intersection with the perpendicular line
3. Then the distance between the given point and the found point is calculated.

Using the properties of perpendicular lines, we find the point of intersection. Let our perpendicular line be: $y_2 = k_2 x + b_2$

$Ax + By + C = 0 \Rightarrow y_1 = \frac{-A}{B}x - \frac{C}{B}$

Using the property of perpendicular lines: $k_1 \cdot k_2 = -1$, we get

$k_1 = \frac{-A}{B} \Rightarrow k_2 = \frac{B}{A}$.

To find b_2 , we use the given point F$(x_0; y_0)$:

So, the equation of the perpendicular line is $y_2 = \frac{B}{A}x + y_0 - \frac{B}{A}x_0$ In the next step, we find the point of intersection of the two lines:

$$y_1 = y_2$$

$$\frac{-A}{B}x - \frac{C}{B} = \frac{B}{A}x + y_0 - \frac{B}{A}x_0 \quad \Rightarrow \quad x = \frac{B^2 x_0 - CA - ABy_0}{A^2 + B^2}$$

$$y = \frac{-B}{A^2 + B^2}(Ax + By + C) + y_0$$

We use the formula for calculating the distance between two points:

$AD = \sqrt{(x - x_0)^2 + (y - y_0)^2}$.

$(x - x_0) = \frac{-A}{A^2+B^2}(\mathrm{A}x_0 + By_0 + C)$; $(y - y_0) = \frac{-B}{A^2+B^2}(\mathrm{A}x_0 + By_0 + C)$

from this, the equation $d = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$ was derived.

Now, if we approach it through mathematical analysis, the shortest distance from a given point to a straight line is the length of the perpendicular drawn to the line. To calculate this distance, we use an analytical-geometric approach and derive a formula through integration. If the function y=f(x) is continuously differentiable on the interval [a,b], then the length of the curve drawn along its graph is calculated using the following formula:

$$l = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx .$$

So, we find the distance by integrating our straight line $y_2 = \frac{B}{A}x + y_0 - \frac{B}{A}x_0$ through the points (x;y) found above and the points given to us $(x_0; y_0)$.

$$\left(\frac{dy_2}{dx}\right) = \frac{B}{A}\ ; \quad l = \int_{x_0}^{x} \sqrt{1+\frac{B^2}{A}}\, dx = \left|\frac{\sqrt{A^2+B^2}}{A}\Big|_{x_0}^{x}\right| = \left|\frac{\sqrt{A^2+B^2}}{A}(x-x_0)\right| = \left|\frac{\sqrt{A^2+B^2}}{A}\frac{-A}{A^2+B^2}(Ax_0 + By_0 + C)\right| = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$$

This distance can be calculated not only mathematically, but also analytically using vectors. The general equation of a straight line is:

ax+by+c=0

Here a and b are the components of the vector normal (perpendicular) to the straight line. Thus, the normal vector is:

$\vec{n} = \begin{bmatrix} a \\ b \end{bmatrix}$

We choose any point on the straight line. For example, if we take x = 0, we find y as follows:

$$y = -\frac{c}{b}$$

Therefore, the point on the line is Q= $(0; -c\ /\ b)$.

The vector from a point to a point on a line is:

$$\overrightarrow{PQ} = \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \end{bmatrix}$$

Where $P(x_0; y_0)$, is the given point, $Q(x_1, y_1)$ is the point on the line.

The distance $\overrightarrow{PQ}$ is the length of the projection of this vector onto the vector $\vec{n}$.The length of the projection is calculated as:

$D = \left|\frac{\overrightarrow{PQ}\cdot\vec{n}}{||\vec{n}||}\right|$

Here:

$\overrightarrow{PQ}\cdot\vec{n}$ is the scalar product,

$||\vec{n}|| = \sqrt{a^2+b^2}$ is the length of the normal vector.

Continuing the calculation: Where:

$$\overrightarrow{PQ}\cdot\vec{n} = a(x_1 - x_0) + b(y_1 - y_0)$$

However, $ax_1 + by_1 + c = 0$, since Q lies on the line. Hence:

$$\overrightarrow{PQ}\cdot\vec{n} = a(x_1 - x_0) + b(y_1 - y_0) = -ax_0 - by_0 - c$$

Thus, the distance:

D=$\frac{|-ax_0-by_0-C|}{\sqrt{A^2+B^2}} = \frac{|ax_0+by_0+C|}{\sqrt{A^2+B^2}}$

Conclusion: Calculating the distance from a point to a straight line is one of the important practical problems of analytical geometry and mathematical analysis. There are several methods for determining this distance, among which the geometric method, approaches based on vector deduction and analysis methods stand out.

Through the geometric approach, the distance is found directly using the classical formula

$$d = \frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}.$$

Each of these methods shows the distance from a different perspective: one is visual-geometric, one is algebraic, and the other is based on variational analysis. This shows that the topic is enriched with theoretical foundations and deserves a comprehensive study.

All methods ultimately give the same result - the above distance formula. This clearly shows that different approaches in mathematics complement each other, confirm each other, and serve for a deeper understanding. Therefore, this topic has not only analytical but also didactic significance, and is one of the important areas of knowledge that connects geometry and mathematical analysis.

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