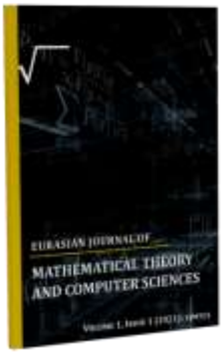

MATHEMATICAL MODELING OF LAMINAR SYMMETRICAL FLOW WITH VISCOSITY

[1]Abduxamidov Sardor Kaxarboyevich
[1]Intern researcher at the Institute of Seismic Strength of Mechanical Structures of the Academy of Sciences of the Republic of Uzbekistan,
[2]Ibrokhimov Abdulfatto Rakhimjon o'g'li
[2]Fergana Polytechnic Institute, Fergana, Uzbekistan
E-mail: ibroximov-2022@mail.ru.
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ABSTRACT

Until now, there are many works dedicated to the drastic simplification of the Nave-Stokes equations and finding their particular solutions that are very relevant for practice. These studies have generally been based on either symmetry considerations or dimensionality and dimension theory. In this article, a mathematical model of the motion of an incompressible fluid in a diffusion nozzle was created and numerical results were obtained. For this, the Nave-Stokes equation was reduced to a parabolic equation with Mises substitutions. The radial and axial velocities of the liquid were determined using the obtained numerical results.

In the world, great attention is being paid to solving current issues based on the hydrodynamic model of the dynamics of the movement of incompressible viscous fluids using scientific and innovative and modern information technologies[1-2]. According to the United Nations, "in the next 15 years, the level of shortage of drinking water for humanity will exceed 40%, the reserves of drinking water on our planet are in a disastrous state, until now, up to 20% of underground water has been used, in this case, in 2050, the population of the earth will be about 9 billion is estimated to reach, as a result of the constant growth of the relative balance of water demand and consumption, including, by 2030, the demand for water for personal and economic purposes is expected to increase by 55%"`[6] In this direction, it is becoming important to create mathematical models that ensure the efficient use of drinking water reserves in the developed countries of the world, including the USA, England, France, Japan, Korea, Germany, the Russian Federation and other countries.

Special attention is paid to mathematical modeling of hydrodynamic systems, research of problems arising in flows in straight channels, boundary layers and pipes, obtaining relevant theoretical and practical results, and applying them in practice. In this regard, including propagation of small shocks in the flow of incompressible viscous fluids, stability of fluid flows in straight channels, boundary layer and pipes, the law of change of hydraulic parameters characterizing the flow, determining the effect of pressure gradient on the main velocity profile of water flows and researching the stability of these flows, the steady field of flows in its to determine the neutral stability curves that separate from the non-stationary region. The solution to the problems of turning chaotic water flows into ordered (laminar) flows and the creation of a device for this activity are important tasks in the effective use and management of water. [3-5]

A number of scientists have created and improved adequate mathematical models describing the behavior of incompressible viscous fluids and their numerical calculation methods: M.A. Lavrentev, B.V. Shabat, L.D. Landau, E.M. Lifshits, S.S. Lin, G. Shlichting, M.A. Goldshtik, V.N. Shtern, F. Drazin, A.N. Tikhonov, N.N. Yanenko, S.K. Godunov, O.A. Ladijenskaya, N.A. Jeltukhin, V.Ya. Levchenko, A.G. Slepsov, A.S. Solovev, F. B. Abutaliev, Ch. B. Normurodov, S. A. Orszag, H. Salwen, C. E. Grosch, D. Gottliev, E. Turkel, A. T. Patera, T. J. Bridges, A. Zebib, etc. developed. [7]

Currently, there are no universal methods designed for comprehensive analysis of incompressible viscous fluid behavior, The problems of approximating the Navier-Stokes equation in the boundary layer at different values of the characteristic parameters and velocity gradient, analyzing the movement of incompressible viscous fluids in the boundary layer and in pipes, determining the critical Reynolds number, and creating mathematical models that allow researching and predicting the processes of transformation of laminar flows into turbulent flows are sufficiently solved. not studied. [8-9]

Mathematical modeling of these complex processes in the movement of fluids is based on the methods of mechanics of the fluid medium. They are usually interpreted in terms of problems related to the Nave-Stokes equations, which describe the behavior of incompressible viscous fluids. The mathematical model describing the behavior of incompressible viscous fluids, the nonstationary three-dimensional Nave-Stokes equations, is presented in the works of G. Shlichting, L. G. Loysyansky, and S. S. Lin. The model has three equations of motion and one continuity equation, with three components of the velocity vector and pressure as unknowns. It is based on the hypothesis that pressure does not depend on time. In the conducted studies, the Nave-Stokes equations were considered in relation to laminar boundary layer problems, and the mechanism of transformation of laminar flow into turbulent flow in the boundary layer was described. The problems of constructing some specific solutions of the Nave-Stokes equations in the boundary layer have been solved. In these studies, the research was carried out on the basis of the asymptotic method. Using the asymptotic method requires very complex calculations, and the resulting asymptotic solutions are crude in appearance. Until now, there are many works [10-11] dedicated to the drastic simplification of the Nave-Stokes equations and finding their special solutions that are very relevant for practice. These studies have generally been based on either symmetry considerations or dimensionality and dimension theory.

In this article, a mathematical model of the motion of an incompressible fluid in a diffusion nozzle was created and numerical results were obtained. For this, the axial and radial velocities of water were determined by the Navier-Stokes equation [12]

Mathematical modeling of incompressible fluid motion in a diffusion nozzle

In order to study the motion of an incompressible fluid in a diffusion nozzle, we will work on the following problem. A view of a diffusion nozzle is illustrated in Figure 1 below.

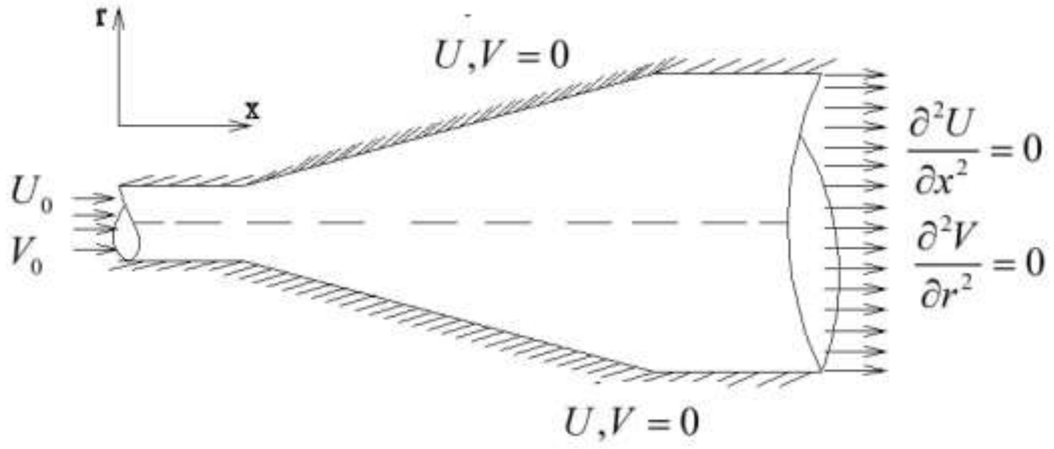


Fig. 1 View of the diffusion nozzle

The Navier-Stokes differential equation and flow continuity equations for incompressible viscous fluids have the following form in the cylindrical coordinate system [1].

$$\begin{cases} \frac{\partial U}{\partial x} + \frac{\partial Vr}{r\partial r} = 0 \\ \frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial r} + \frac{1\partial P}{\rho\partial x} = \nu\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial r^2} + \frac{\partial U}{r\partial r}\right) \\ \frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial r} + \frac{1\partial P}{\rho\partial r} = \nu\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial r^2} + \frac{\partial V}{r\partial r} - \frac{V}{r^2}\right) \end{cases} \quad (1)$$

Here U,V - axial and radial velocity of water in the channel, P - hydrostatic pressure, ρ - the density of the liquid $(\rho = const)$, t - time, x,r - cylindrical coordinate system.

We introduce the stream function to solve the Navier-Stokes system of equations. $U = \frac{\partial\psi}{r\partial r};\quad V = -\frac{\partial\psi}{r\partial x};\quad \Im = \frac{\partial U}{\partial r} - \frac{\partial V}{\partial x}$

(1) After we introduce the stream function in the equation, the pressure decreases and the equation appears

$$\begin{cases} \frac{\partial\Im}{\partial t} + U\frac{\partial\Im}{\partial x} + V\frac{\partial\Im}{\partial r} - \frac{V\Im}{r} = \nu\left(\frac{\partial^2\Im}{\partial x^2} + \frac{\partial^2\Im}{\partial r^2} + \frac{\partial\Im}{r\partial r} - \frac{\Im}{r^2}\right) \\ \frac{\partial}{\partial r}\left(\frac{\partial\psi}{r\partial r}\right) + \frac{\partial^2\psi}{r\partial x^2} = \Im \end{cases} \quad (2)$$

(2) from Equation $\Im = 0$ out of necessity ψ the value of the function is found

$$-\frac{1}{r^2}\frac{\partial\psi}{\partial r} + \frac{1}{r}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial^2\psi}{\partial x^2} = 0$$

$$\frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{\partial^2\psi}{\partial x^2} = 0 \quad (3)$$

To numerically solve system (3) using Mises substitutions $(x,r) \to (\xi,\eta)$ coordinate substitutions are performed

$$\xi = x,\ \eta = \frac{r}{f},\ f = 1 + 2arctgx,\ f' = \frac{2}{1+x^2}$$

$$\frac{\partial}{\partial x} = \frac{\partial \xi}{\partial x}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x}\frac{\partial}{\partial \eta} \qquad \frac{\partial \xi}{\partial x} = 1,\ \frac{\partial \eta}{\partial x} = \frac{rf'}{f^2} = A,\ \ \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi} - \frac{rf'}{f^2}\frac{\partial}{\partial \eta}$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2} - 2\frac{rf'}{f^2}\frac{\partial^2}{\partial \xi \partial \eta} + \left(\frac{rf'}{f^2}\right)^2 \frac{\partial^2}{\partial \eta^2} - \left(\frac{rf'}{f^2}\right)' \frac{\partial \psi}{\partial \eta}$$

$$\frac{\partial}{\partial r} = \frac{\partial \xi}{\partial r}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial r}\frac{\partial}{\partial \eta} \qquad \frac{\partial \xi}{\partial r} = 0,\ \frac{\partial \eta}{\partial r} = \frac{1}{f},\ \ \frac{\partial}{\partial r} = \frac{1}{f}\frac{\partial}{\partial \eta}$$

$$\frac{\partial^2}{\partial r^2} = \frac{1}{f^2}\frac{\partial^2}{\partial \eta^2}$$

Equation (3) after Mises substitutions, the equation becomes the following

$$\frac{\partial \psi}{\partial t} + \frac{\partial^2 \psi}{\partial \xi^2} + \frac{1}{f^2}\frac{\partial^2 \psi}{\partial \eta^2} - \frac{1}{rf}\frac{\partial \psi}{\partial \eta} - 2\frac{rf'}{f^2}\frac{\partial^2 \psi}{\partial \eta \partial \xi} + \left(\frac{rf'}{f^2}\right)' \frac{\partial \psi}{\partial \eta} + \left(\frac{rf'}{f^2}\right)^2 \frac{\partial^2 \psi}{\partial \eta^2} = 0 \qquad (4)$$

From this $A = \frac{rf'}{f^2};\ B = \left(\frac{rf'}{f^2}\right)^2;\ C = \left(\frac{rf'}{f^2}\right)'$ we can determine.

For this, we write equation (4) in the following form

$$\frac{\psi_{i,j}^{n+1} - \psi_{i,j}^{n}}{\Delta t} = \frac{\psi_{i+1,j}^{n} - 2\psi_{i,j}^{n} + \psi_{i-1,j}^{n}}{\Delta \xi^2} + \left(B + \frac{1}{f^2}\right)\frac{\psi_{i,j+1}^{n} - 2\psi_{i,j}^{n} + \psi_{i,j-1}^{n}}{\Delta \eta^2} -$$

$$-\left(\frac{1}{rf} - C\right)\frac{\psi_{i,j+1}^{n} - \psi_{i,j-1}^{n}}{2\Delta \eta} - 2A\frac{\psi_{i+1,j+1}^{n} - \psi_{i+1,j-1}^{n} - \psi_{i-1,j+1}^{n} + \psi_{i-1,j-1}^{n}}{4\Delta \xi \Delta \eta}\frac{\partial^2 \psi}{\partial \eta \partial \xi} \qquad (5)$$

This is in the numerical solution of equation (4) ψ is found and the initial radial and radial velocities are found and the cumulative function $\Im$ real axial and radial velocities are found by iteration method.

Initial and boundary conditions

Entering $\frac{\psi}{\psi_0} = 1.$

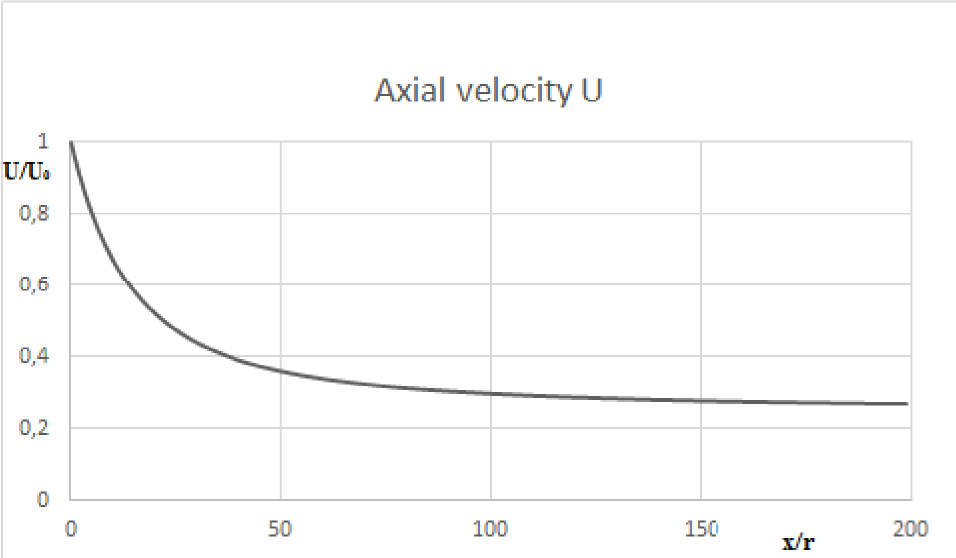
From the condition of viscosity at the wall $U = 0,\ \ V = 0.$

An extrapolation condition was used for all velocities in the output.

$$\frac{\partial^2 U}{\partial x^2} = 0,\ \frac{\partial^2 V}{\partial r^2} = 0.$$

Analysis of numerical results

Figure 2 below shows the axial and radial velocity graphs of the flow in the nozzle.



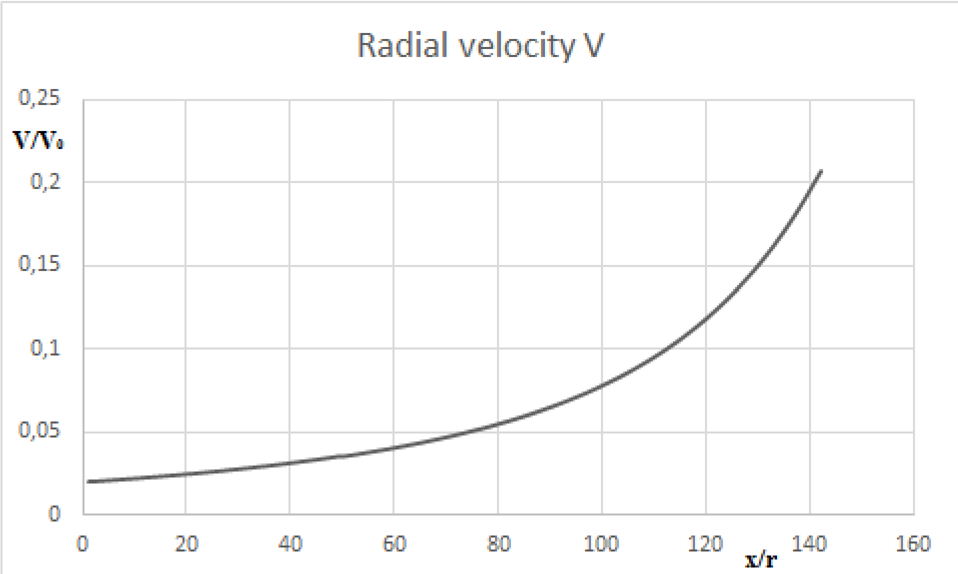


Fig. 2 Graphs constructed based on numerical results of axial and radial velocities of the flow in the nozzle

Conclusion

In this paper, the motion of an incompressible viscous fluid in a diffusion-symmetric nozzle is studied. The stream function was used to calculate the Navier-Stokes equation to determine the flow velocities. Reynolds number $\mathrm{Re} = 1000$,time $\Delta t = 0.001s$ and the following $\Delta\xi = 0.025$ $\Delta\eta = 0.01$ parameters are accepted. Using a numerical method, numerical results were obtained for radial and radial velocities and graphs were constructed.

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