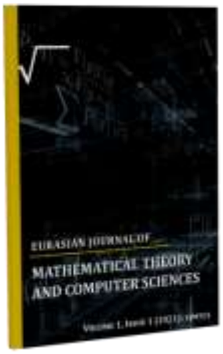


DETERMINANTLAR NAZARIYASINI ANALITIK GEOMETRIYA MASALALARIGA TADBIQI

[1]**Berdiyorov Azamat**
Oliy matematika kafedrasi dotsenti
e-mail: berdiyorov57@gmail.com,
[2]**Arslanov Dostonbek**
Talaba ,Jizzax politexnika institute.
https://doi.org/10.5281/zenodo.7359403

ARTICLE INFO

Received: 15th November 2022
Accepted: 24th November 2022
Online: 25th November 2022

KEY WORDS

Determinant, permutatsiya, faktorial, yuza, hajm, piramida, tekislik, to‘gri chiziq.

ABSTRACT

Ko‘rilayotgan maqolada chiziqli algebra elementlaridan ya’ni ikkinchi va uchinchi tartibli determinantni hisoblash formulasidan foydalanib analitik geometriya masalalaridan bir nechta namunalar ko‘rib chiqilgan.

Determinantlar bizga elementar algebradan ma’lum bo‘lgan permutatsiya (o‘rin almashtirishlar) bilan bog‘liqdir. Istalgan n ta musbat butun sonni, masalan 1 dan n gacha bo‘lgan 1, 2, 3, 4….n sonlarni olamiz. Bu sonlardan hammmasi bo‘lib

$$n! = 1\cdot 2\cdot 3\cdot \ldots \cdot n$$

ta har xil permutatsiya (o‘rin almashtirishlar) tuzish mumkin. Masalan, 1, 2, 3 sonlardan hammasi bo‘lib,

$$3! = 1\cdot 2\cdot 3 = 6$$

ta har xil permutatsiya tuzamiz: 123, 132, 213, 231, 312, 321.

Ta’rif: n-tartibli determinant deb, $n!$ hadlardan tuzilgan algebraik yig‘indiga aytiladi. Bunda yig‘indining har bir hadi determinantdagi n ta elementning ko‘paytmasidan iborat. Agar hadlarning hammasida elementlarning birinchi n raqamlari tartib bilan (ya’ni 1, 2, 3…n) yozilsa, ikkinchi raqamlari 1, 2, 3…n raqamlardan tuziladigan $n!$ ta permutasiya tashkil etadi.

Analitik geometriya masalalarini yechishda quyidagi

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

ikkinchi tartibli determinant va

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - c_1b_2a_3 - b_1a_2c_3 - a_1c_2b_3$$

uchinchi tartibli determinantni hisoblash formulalaridan foydalanamiz.

Bu formulalar yordamida bir nechta masalalarni ko‘rib chiqamiz.

1-masala: A(-3;8) va B(5;-1) nuqtalar orqali o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing hamda shu to‘g‘ri chiziqda absissiyasi 1 ga teng bo‘lgan nuqtani toping.

Yechish: Ikki nuqta orqali o‘tuvchi to‘g‘ri chiziq tenglamasini quyidagi formulalar orqali topamiz.

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Demak

$$\begin{vmatrix} x & y & 1 \\ -3 & 8 & 1 \\ 5 & -1 & 1 \end{vmatrix} = 0$$

$$8x + 5y + 3 - 40 + 3y + x = 0$$

Ixchamlasak,

$$9x + 8y - 37 = 0$$

bo‘ladi.

Bu to‘g‘ri chiziqda absissasi 1 ga teng bo‘lgan nuqtani koordinatasini topish uchun uch nuqtani bir to‘g‘ri chiziqda yotish shartidan foydalanamiz. Ya’ni,

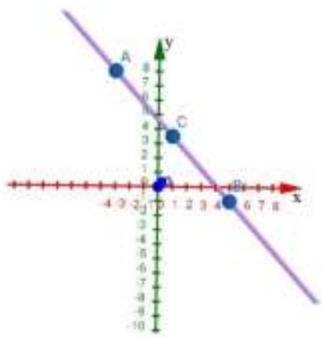
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

U holda,

$$\begin{vmatrix} -3 & 8 & 1 \\ 5 & -1 & 1 \\ 1 & y & 1 \end{vmatrix} = 0 \quad 3 + 8 + 5y + 1 - 40 + 3y = 0 \quad \text{yoki} \quad 8y - 28 = 0 \quad y = 3.5$$

bo‘ladi.

Demak, $9x + 8y - 37 = 0$ to‘g‘ri chiziqda absissasi 1 ga teng bo‘lgan nuqta C(1;3.5) bo‘ladi.(1-rasm.)



1-rasm.

2-masala. Koordinatalar boshidan to‘g‘ri chiziqqa tushirilgan OP perpendikular OX o‘qi bilan $\alpha = \dfrac{\pi}{6}$ burchak hosil qiladi. Agar perpendikular uzunligi 3 bo‘lsa, to‘g‘ri chiziqning normal tenglamasini tuzing.

Yechish: To‘g‘ri chiziqning normal tenglamasini tuzish uchun quyidagi formuladan foydalanamiz:

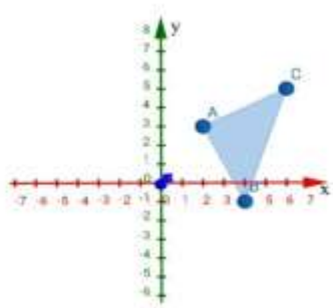
$$\begin{vmatrix} x & -\sin\alpha \\ y & cos\alpha \end{vmatrix} = p$$

Ya'ni,

$$\begin{vmatrix} x & -\sin\frac{\pi}{6} \\ y & \cos\frac{\pi}{6} \end{vmatrix} = 3 \quad \text{yoki} \quad \begin{vmatrix} x & -\frac{1}{2} \\ y & \frac{\sqrt{3}}{2} \end{vmatrix} = 3$$

bu ikkinchi tartibli determinantni hisoblab, $\frac{\sqrt{3}}{2}x + \frac{1}{2}y = 3$ yoki $\sqrt{3}x + y - 6 = 0$ tenglamaga ega bo'lamiz.

3-masala: Uchlari A(2;3), B(4; -1) va C(6;5) bo'lgan uchburchak yuzini hisoblang. (2-rasm.)



2-rasm.

Yechish: Uchburchakning yuzini hisoblash uchun

$$S = \frac{1}{2} mod \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

formuladan foydalanib, quyidagicha hisoblashlarni amalga oshiramiz:

$$S = \frac{1}{2} mod \begin{vmatrix} 2 & 3 & 1 \\ 4 & -1 & 1 \\ 6 & 5 & 1 \end{vmatrix} = \frac{1}{2} mod\left(-2+18+20+6-12-10\right) = \frac{1}{2}\cdot 20 = 10\, kv.birlik$$

4-masala: $M_1(3;-1;2)$, $M_2(4;-1;-1)$, $M_3(2;0;2)$ nuqtalardan o'tuvchi tekislik tenglamasini tuzing.

Yechish: Berilgan uchta nuqtadan o'tuvchi tekislik tenglamasini tuzish uchun

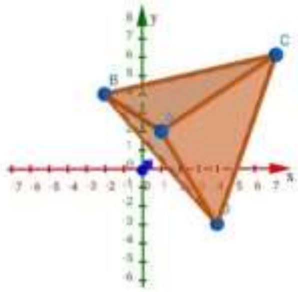
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

formuladan foydalanamiz. U holda

$$\begin{vmatrix} x-3 & y+1 & z-2 \\ 4-3 & -1+1 & -1-2 \\ 2-3 & 0+1 & 2-2 \end{vmatrix}=0 \quad \text{yoki} \quad \begin{vmatrix} x-3 & y+1 & z-2 \\ 1 & 0 & -3 \\ -1 & 1 & 0 \end{vmatrix}=0$$

Bundan $0+3(y+1)+z-2-0+0+3(x-3)=0$ yoki $3x+3y+z-8=0$ tenglamaga ega bo'lamiz.

5-masala. Uchlari A(1;2;3), B(-2;4;1), C(7;6;3) va D(4; -3; -1) nuqtalarda bo'lgan piramida berilgan. a) ABC yog'ining yuzi; b) Piramidaning hajmini toping.



3-rasm.

Yechish: a) ABC yog'ining yuzi $\overrightarrow{AB}$ va $\overrightarrow{AC}$ vektorlarning vektor ko'paytmasi modulini yarmiga teng, ya'ni

$$S_{ABC}=\frac{1}{2}\cdot\left|\overrightarrow{AB}\cdot\overrightarrow{AC}\right|=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

formula orqali topiladi.

$$\overrightarrow{AB}=\{-3;2;-2\}=-3i+2j-2k$$
$$\overrightarrow{AC}=\{6;4;0\}=6i+4j+0k$$

$$\left|\overrightarrow{AB}\cdot\overrightarrow{AC}\right|=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -3 & 2 & -2 \\ 6 & 4 & 0 \end{vmatrix}=-12\vec{j}-12\vec{k}-12\vec{k}+8\vec{i}=8\vec{i}-12\vec{j}-24\vec{k}$$

Demak

$$S_{ABC}=\frac{1}{2}\left|\overrightarrow{AB}\cdot\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{8^2+(-12)^2+(-24)^2}=\frac{1}{2}\sqrt{64+144+576}=\frac{1}{2}\sqrt{784}=\frac{1}{2}\cdot 28=14$$ kv.birlik

b) Piramidaning hajmini topish uchun uch vektorning aralash ko'paytmasidan foydalanamiz. Ya'ni,

$$V_{pir}=\frac{1}{6}mod\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

formula yordamida topamiz.

$$\overrightarrow{AB} = -3\vec{i} + 2\vec{j} - 2\vec{k}$$
$$\overrightarrow{AC} = 6\vec{i} + 4\vec{j} + 0\vec{k}$$
$$\overrightarrow{AD} = 3\vec{i} - 5\vec{j} - 4\vec{k}$$

Ekanligini hisobga olsak, V_{pir} ko‘rinishi

$$V_{pir} = \frac{1}{6}\mathrm{mod}\begin{vmatrix} -3 & 2 & -2 \\ 6 & 4 & 0 \\ 3 & -5 & -4 \end{vmatrix} = \frac{1}{6}\mathrm{mod}(48+60+24+48) = \frac{1}{6}\mathrm{mod}(180) = 30$$

kub birlik ga ega bo‘lamiz. Demak piramida hajmi 30 kub birlikka teng ekan.

References:

1. T.Shodiyev. Analitik geometriyadan qo‘llanma. Oliy texnik o‘quv yurtlari uchun T.: “O‘qituvchi” 1973.

2. Д.В.Клетеник. Сборник задач по аналитической геометрии. Наука 2003 Москва.

3. А.Ш.Бердиёров, Я. Усанов, Х.Жабборов “Айланага доир баъзи мисол ва масалаларни ечиш усуллари ҳақида ФМИ 5-сон 2019 йил.

4. ТТ А. Ш. Б., ТТ У. Я. Т., Хайдаров Т. Т. Построение периодических решений квазилинейных уравнений при резонансе в критическом случае первого порядка //Инновационная наука. – 2015. – №. 5-3. – С. 10-15.

5. Бердияров А. Ш. и др. О существовании периодических решений линейных интегро-дифференциальных уравнений типа Вольтерра с последействием //Ученый XXI века. – 2016. – Т. 3.

6. Бердиёров А. Ш., Тураев У. Я., Жабборов Х. Х. ОБ УСЛОВИЯХ СУЩЕСТВОВАНИЯ ПЕРИОДИЧЕСКИХ РЕШЕНИЙ ИНТЕГРОДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ.

7. Soatov, U. A. (2018). Djonuzoqov. UA" Problems of geometry with the help of joint application of basic theorems and formulas". Scientific-methodical journal of" Physics, Mathematics and Informatics", (4), 40.

8. Soatov Ulugbek Abdukadirovich, & Dzhonuzokov Ulugbek Abduganievich (2020). ABOUT THE ISSUES OF GEOMETRICAL INEQUALITIES AND THE METHODS OF THEIR SOLUTION. European science, (7 (56)), 5-10.

9. Soatov, U. A. (2022). Tenglamalarni yechishning grafik usuli haqida. Science and Education, 3(8), 7-12.

10. Soatov U.A., Axadova K.S. TENGLAMA VA TENGSIZLIKLARNI YECHISHNING BA’ZI NOSTANDART USULLARI HAQIDA//Tafakkur ziyosi ilmiy-uslubiy jurnali. -2021/2-son. - C. 118-122.