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ALGEBRAIC REPRESENTATION OF COMPLEX NUMBERS

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ABSTRACT

We know that square of a real number is always non-negative e.g. $(4)^2 = 16$ *and* $(-4)^2 = 16$ *. Therefore, square root of* 16 *is* ± 4*. What about the square root of a negative number? It is clear that a negative number can not have a real square root. So we need to extend the system of real numbers to a system in which we can find out the square roots of negative numbers. Euler (1707-1783) was the first mathematician to introduce the symbol* i *(iota) for positive square root of* -1 *i.e. ,* $i = \sqrt{-1}$.

Introduction Solving algebraic equations has been historically one of the favorite topics of mathematicians. While linear equations are always solvable in real numbers, not all quadratic equations have this property. The simplest such equation is $x^2 + 1 = 0$. Until the 18th century, mathematicians avoided quadratic equations that were not solvable over □ . Leonhard Euler broke the ice introducing the "number' $\sqrt{-1}$ in gis famous book Elements of Algebra as ". . . neither nothing, nor greater than nothing, nor less than nothing . . ." and observe ". . . notwithstanding this, these numbers present themselves to the mind; they exist in our imagination and we still have a sufficient idea of them; . . . nothing prevents us from making use of these imaginary numbers, and employing them in calculation" . Euler denoted the number $\sqrt{-1}$ by i and called it the imaginary unit. This became one of the most useful symbols in mathematics. Using this symbol one defines complex numbers as $z = a + bi$, where a and b are real numbers. The study of complex numbers continues and has been enhanced in the last two and a half centuries; in fact, it is impossible to imagine modern mathematics without complex numbers. All mathematical domains make use of them in some way. This is true of other disciplines as well: for example, mechanics, theoretical physics, hydrodynamics, and chemistry.

Main Part

Imaginary numbers: Square root of a negative number is called an imaginary number., for exaple,

$$\sqrt{-9} = \sqrt{-1}\sqrt{9} = 3i,$$

Integral powers of I :

$i=\sqrt{-1}$, $i^2=-1$, $i^3=i^2i=-i$, $i^4=(i^2)^2=(-1)^2=1$

To compute i^n for $n>4$, we divide n by 4 and write it in the form $n=4m+r$, where m is quotient and r is remainder $(0\le r\le 4)$

Hence $i^n=i^{4m+r}=(i^4)^m\cdot(i)^r=(1)^m(i)^r=i^r$

For example, $i^{39}=i^{4\times 9+3}=(i^4)^9\cdot(i)^3=(i)^3=-i$

And $(i)^{-435}=i^{-(4\times 108+3)}=(i\ )^{-(4\times 108)}\cdot(i)^{-3}=\frac{1}{(i^4)^{108}}\cdot\frac{1}{(i)^3}=\frac{1}{(i)^4}=i$

If a and b are positive real numbers,then

$\sqrt{-a}\times\sqrt{-b}=\sqrt{-1}\sqrt{a}\times\sqrt{-1}\sqrt{b}=i\sqrt{a}\times i\sqrt{b}=-\sqrt{ab}$

$\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$ if a and b are positive or at least one of is negative or zero. However, $\sqrt{a}\sqrt{b}\ne\sqrt{ab}$ if a and b, both are negative.

Complex numbers:

A number which can be written in the form $a+ib$, where a ,b are real numbers and $i=\sqrt{-1}$ is called a complex number.

If $z=a+bi$ is the complex number,then a and b are called real and imaginary part, respectively, of the complex number and written as $\mathrm{Re}(z)=a$, $\mathrm{Im}(z)=b$.

Order relations " greater than" and " less than" are not defined for complex numbers.

If the imaginary part of a complex number is zero, then the complex number is known as purely real number and if real part is zero, then it is called purely imaginary number,for example, 2 is a purely real number because its imaginary part is zero and $3i$ is a purely imaginary number because its real part is zero

Algebra of complex numbers:

Two complex numbers $z_1=a+ib$ and $z_2=c+id$ are said to be equal if $a=c$ and $b=d$.

Let $z_1=a+ib$ and $z_2=c+id$ be two complex numbers then $z_1+z_2=(a+c)+i(b+d)$.

Multiplication of complex numbers:

Let $z_1=a+ib$ and $z_2=a+id$, be two complex numbers. Then $z_1\cdot z_2=(a+ib)(c+id)=(ac-bd)+i(ad+bc)$

1. As the product of two complex numbers is a complex numbers, the set of complex numbers is closed with respect to multiplication.
2. Multiplication of complex numbers is commutative, i.e., $z_1\cdot z_2=z_2\cdot z_1$
3. Multiplication of complex numbers is associative,i,e., $(z_1\cdot z_2)\cdot z_3=z_1\cdot(z_2\cdot z_3)$

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4. For any complex number $z = x + iy$, there exists a complex number 1,i.e., $(1+0i)$ such that $z \cdot 1 = 1 \cdot z = z$, known as identity element for multiplication.

5. For any non zero complex number $z = x + iy$, there exists a complex number $\frac{1}{z}$ such that $z \cdot \frac{1}{z} = \frac{1}{z} \cdot z = 1$,i.e., multiplicative inverse of $a + ib = \frac{1}{a+ib} = \frac{a-ib}{a^2+b^2}$.

6. For any three complex numbers z_1, z_2 and z_3,

$$z_1 \cdot (z_2 + z_3) = z_1 \cdot z_2 + z_1 \cdot z_3$$

And $$(z_1 + z_2) \cdot z_3 = z_1 \cdot z_3 + z_2 \cdot z_3$$

Conjugate of a complex number:

Let $z = a + bi$ be a complex number. Then a complex number obtained by changing the sign of imaginary part of the complex number is called the conjugate of z and it is denoted by $\bar{z}$, i.e., $\bar{z} = a - ib$.

Note that additive inverse of z is $-a - ib$ but conjugate of z is $-a - ib$.

We have:

1. $\overline{(\bar{z})} = z$
2. $z + \bar{z} = 2\operatorname{Re}(z), z - \bar{z} = 2i\operatorname{Im}(z)$
3. $z = \bar{z}$, if z is purely real.
4. $z + \bar{z} = 0 \Leftrightarrow z$ is purely imaginary
5. $z \cdot \bar{z} = \{\operatorname{Re}(z)\}^2 + \{\operatorname{Im}(z)\}^2$
6. $\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$, $\overline{(z_1 - z_2)} = \bar{z}_1 - \bar{z}_2$
7. $\overline{(z_1 \cdot z_2)} = \overline{(z_1)}\overline{(z_2)}$, $\overline{\frac{z_1}{z_2}} = \frac{\overline{(z_1)}}{\overline{(z_2)}} (\overline{z_2} \neq 0)$

Modulus of a complex number:

Let $z = a + bi$ be a complex number. Then the positive square root of the sum of square of real part and square of imaginary part is called modulus(absolute value) of z and it is denoted by $|z|$ i.e., $|z| = \sqrt{a^2 + b^2}$

In the set of complex numbers $z_1 \rangle z_2$ or $z_1 \langle z_2$ ar meaningless but

$$|z_1| \rangle |z_2| \quad \text{or} \quad |z_1| \langle |z_2|$$

Are meaning because $|z_1|$ and $|z_2|$ are real number.

Properties of modulus of a complex number:

1. $|z| = 0 \Leftrightarrow z = 0$ i.e., $\operatorname{Re}(z) = 0$ and $\operatorname{Im}(z) = 0$

2. $|z| = |\bar{z}| = |-z|$

3. $-|z| \le \operatorname{Re}(z) \le |z|$ and $-|z| \le \operatorname{Im}(z) \le |z|$

4. $z\bar{z} = |z|^2, |z^2| = |\bar{z}|^2$

5. $|z_1 z_2| = |z_1| \cdot |z_2|, \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} (z_2 \ne 0)$

6. $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2\operatorname{Re}(z_1 \overline{z_2})$

7. $|z_1 - z_2|^2 = |z_1|^2 + |z_2|^2 - 2\operatorname{Re}(z_1 \overline{z_2})$

8. $|z_1 + z_2| \le |z_1| + |z_2|$

9. $|z_1 - z_2| \ge |z_1| - |z_2|$

10. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

In particular:

$|z_1 - z_2|^2 + |z_1 + z_2|^2 = 2(|z_1|^2 + |z_2|^2)$

11. As stated earlier multicative inverse (reciprocal) of a complex number $z = a + bi(\ne 0)$ is $\frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\bar{z}}{|z|^2}$

Conclusion

Complex analysis is one of the most beautiful fields of mathematics. It has numerous connections with the most branches of pure and applied mathematics: algebraic geometry, number theory, physics(electrostatics, hydrodynamics, heat conduction), probability, combinatorics.

Historically, complex numbers were introduced in 16th century as a way to interpret the Cardano formula for the roots of cubic polynomial $x^3 + px + q = 0$;

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

To obtain all 3 roots of the cubic equation we have to interpret the cubic root as a multivalued function with values in □ .

For several hundred years after Cardano complex numbers remained an obscure topic. It took several centuries and efforts of the best mathematicians of their time to demonstrate the fascinating nature of complex numbers and complex analysis,and build the ground for modern applications.

References:

1. Titu Andreescu, Dorin Andrice .Complex Numbers from A to…Z. Birkhauser,2004
2. James Ward Brown, Ruel V. Churchill. COMPLEX VARIABLES AND APPLICATIONS,2004
3. Chirstophre C. Tisdell & bookboon.com. Introduction to Complex Numbers: YouTube Workbook 1stedition,2015
4. Adler, I., A New Look at Geometry, John Day, New York,1966