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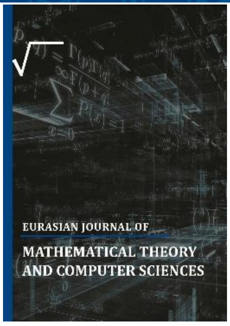

NEW ALGORITHMIC SOLUTIONS AND THEIR MATHEMATICAL FOUNDATIONS

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ABSTRACT
This article examines new algorithmic solutions and their underlying mathematical foundations. It analyzes complex mathematical principles necessary for understanding and developing advanced algorithms in areas such as machine learning, quantum computing, blockchain technology, optimization, and graph theory. The article highlights the role of linear algebra, calculus, probability theory, and information theory in modern algorithms. It also discusses mathematical approaches to pressing issues such as algorithmic fairness and interpretability. By synthesizing existing research and conceptual ideas, the article demonstrates the inseparable connection between abstract mathematical theory and practical algorithmic innovations, and outlines future research and application directions.

Introduction. The rapid development of modern technologies has significantly increased the demand for new algorithmic solutions. Artificial intelligence, big data analysis, quantum computing, and blockchain require the application of deep mathematical concepts to algorithmic thinking. Algorithms are not just a set of calculation steps, but rather a central mechanism that determines the operation of complex systems, processes data, and makes decisions. This article is devoted to the theoretical and practical foundations of new algorithmic solutions, emphasizing the importance of the mathematical principles and models underlying them. A deep understanding of the mathematical foundations is necessary not only to understand the limitations of existing algorithms, but also to create completely new, more efficient, and reliable solutions. In this article, we will examine the mathematical foundations of several key areas, including machine learning, quantum and blockchain technologies, optimization and graph algorithms, as well as algorithmic fairness and interpretability.

Analysis of relevant literature

In analyzing the mathematical foundations of new algorithmic solutions, sources related to the field of deep learning are of particular importance among the existing literature. For example, the supplement "Mathematics for Deep Learning" is designed to provide the mathematical background necessary for understanding the basic theory of deep learning. This source explains in detail how fundamental mathematical concepts such as linear algebra, differential and integral calculus, probability theory, statistics, and information theory are used in deep learning models [1]. Although the authors do not always require a complete theoretical understanding, they emphasize that solving real-world problems such as gradient flow, hidden assumptions in loss functions, or entropy requires a deeper understanding of mathematical concepts.

Benoit Liquet, Sarat Moka, and Yoni Nazarathy's book Mathematical Engineering of Deep Learning (CRC Press, 2024) provides a comprehensive and concise mathematical explanation [2]. This publication covers fundamental concepts such as linear algebra, probability theory, and optimization, as well as convolutional neural networks (CNN), recurrent neural networks (RNN), transformers, generative adversarial networks (GANs), diffusion models, reinforcement learning, and graph- . The authors focus on the fundamental mathematical definitions of deep learning models, algorithms, and techniques, while also discussing computer codes, neural networks, historical perspectives, and theoretical research. This approach allows specialists in fields such as engineering, signal processing, statistics, and physics to quickly grasp the basic mathematical components of modern deep learning. In addition, the book includes a "mathematics crash course" for learning elementary mathematical concepts which helps non-mathematicians understand concepts such as sigmoidal and softmax networks in the context of deep learning models, as well as vectors, matrices, and gradients [2].

Other sources include metadata information on deep learning algorithms [3] or separate algorithmic concepts such as the result of a binary tree search operation [4]. Although this information indicates the existence of online resources on deep learning mathematics, it does not provide a comprehensive theoretical analysis of the broader mathematical foundations of algorithmic solutions. In general, the existing literature clearly demonstrates the importance of mathematical foundations for new algorithmic paradigms such as deep learning and the deep attention paid to them. Nevertheless, there is a need for a synthesis of literature that provides a comprehensive analysis of the mathematical foundations of other promising algorithmic approaches, such as quantum computing, blockchain technology, and algorithmic fairness.

Research methodology

This article is based on a conceptual analysis and synthesis of literature on new algorithmic approaches and their mathematical foundations. The research consists of several main stages. First, a comprehensive literature review was conducted to identify the most important achievements in the fields of modern algorithmic paradigms, namely machine learning, quantum computing, blockchain technology, optimization, and graph theory. Second, the mathematical principles and theories underlying these algorithms, including linear algebra, differential calculus, probability theory, information theory, cryptography, and discrete mathematics, were studied. Thirdly, efforts were made to systematically identify the relationship between the mathematical models relevant to each algorithmic solution and their practical application. Fourth, the role of mathematical approaches in solving complex problems such as algorithmic fairness and comprehensibility was analyzed. The information presented in the article is mainly taken from academic publications, books, and scientific articles, reflecting the latest knowledge in this field. This methodology allows for the formation of a complex understanding of various algorithmic domains and demonstrates the general significance of their mathematical foundations.

Machine learning (ML) algorithms occupy a central place among modern algorithmic solutions, and their effectiveness is based on deep mathematical concepts. Several fundamental mathematical disciplines are essential for understanding and optimizing the working mechanism of ML models.

Linear algebra: In MO, data is often presented in the form of vectors and matrices. Linear algebra concepts, such as vector spaces, matrix multiplication, eigenvalues and eigenvectors, data transformation, reduction (e.g., principal component analysis – PCA), and modeling signal propagation in neural networks [1]. Golib algebra also plays a key role in controlling the weights and biases of deep learning networks [2].

Differential calculus: Gradient-based optimization algorithms, such as gradient descent and its variants, are the basis for optimizing model weights in MO ( ). Differential calculus, especially for functions with many variables, allows us to determine the direction of minimizing the loss function through partial derivatives and gradient estimates [1]. The backpropagation algorithm uses chain rules to efficiently calculate gradients in deep neural networks.

Probability theory and statistics: These fields provide the ability to make decisions based on the uncertainty of MO models. Random variables, probability distributions (e.g., normal, Bernoulli), Bayes' theorem, and maximum likelihood estimation form the theoretical basis of classification, regression, and clustering algorithms [1]. In fact, many deep learning models, such as generative adversarial networks (GANs) and diffusion models, are based on probability models [2].

Information theory: Entropy, mutual information, and Kullback-Leibler divergence are used to quantitatively evaluate how well MO models store, transmit, and process information [1]. They play an important role in shaping loss functions (e.g., cross-entropy) and compressing data.

Optimization: In addition to the gradient descent mentioned above, optimization theory includes more complex techniques such as convex optimization, subgradients, and constrained optimization, which are widely used in training and tuning complex MO models.

These mathematical foundations ensure not only the practical effectiveness of machine learning algorithms, but also their theoretical soundness, which creates a basis for the rapid development of the field.

Quantum computing and blockchain technology are the most promising algorithmic paradigms of the 21st century, based on completely new mathematical models and concepts.

Quantum algorithms: Quantum computing uses fundamental principles of quantum mechanics, such as superposition, entanglement, and quantum interference. These phenomena enable the creation of new types of algorithms that can solve problems that are impossible for classical computers. The mathematical basis of quantum algorithms is based on complex linear algebra and quantum mechanics. Quantum bits (qubits) are represented as states in complex vector spaces (Hilbert spaces), and quantum gates are defined as unitary matrices that act on these states. Wellknown quantum algorithms such as the Shor algorithm (for factoring large numbers) and the Grover algorithm (for searching unstructured databases) demonstrate exponential or quadratic speedup over classical algorithms using this mathematical apparatus.

Blockchain algorithms: Blockchain technology combines a number of cryptographic and algorithmic principles to create a decentralized, immutable, and transparent ledger. Its mathematical foundations are based on cryptography, discrete mathematics, and graph theory. The main mathematical concepts are as follows:

- Hash functions: One-way mathematical functions used to ensure the integrity of data and link blocks together. They form the basis of security and immutability.
- Asymmetric cryptography (public key cryptography): Used to ensure the authenticity of digital signatures and transactions, it allows each participant to have their own private (secret) and public (open) key pair.
- Consensus mechanisms (e.g., Proof-of-Work): Algorithms used to reach agreement among participants in the network on the state of the ledger ( ). These mechanisms are based on probability theory and game theory.

Quantum and blockchain algorithms not only promise technological breakthroughs, but also open up new directions for mathematical research, which creates a solid theoretical foundation for future algorithmic solutions.

Optimization and graph algorithms are widely used not only in traditional areas of computer science, but also in modern complex systems. Their mathematical foundations allow them to effectively solve various problems.

Optimization algorithms: The goal of optimization is to find the maximum or minimum value of an objective function under certain constraints. Modern optimization algorithms include the following:

- Convex optimization: Designed to work with convex sets and convex functions, it is used in many machine learning models (e.g., Support Vector Machines) and in resource allocation. Its mathematical basis is based on convex analysis.
- Gradient-based optimization variants: Algorithms such as stochastic gradient descent (SGD), Adam, and RMSProp, which are widely used in machine learning, are optimized versions of gradient descent for working with large amounts of data. They are based on differential calculus and are designed to minimize the loss function according to model parameters [1].

Evolutionary and metaheuristic optimization: Methods such as genetic algorithms, ant colony optimization, and particle swarm optimization are used to solve complex, non-convex problems that cannot be solved using traditional gradient-based methods. These methods are based on probability theory, combinatorics, and metaphors inspired by nature.

Graph algorithms: Graph theory provides a powerful framework for modeling objects (nodes) and the relationships between them (edges). Graph algorithms play an important role in analyzing complex networks such as the Internet, social networks, transportation systems, and biological networks.

- Shortest path algorithms: Dijkstra, Bellman-Ford, and Floyd-Warshall algorithms are used in navigation systems, network routing, and logistics to find the shortest paths. They are based on discrete mathematics and dynamic programming principles.
- Minimum spanning tree algorithms: Kruskal and Prim algorithms are used to find the minimum cost network in network construction (e.g., telecommunications, electrical networks).
- Network flow algorithms: Maximum flow (max-flow) and minimum cut (mincut) problems are used to allocate resources, distribute loads, and optimize network capacity.
- Graph neural networks (GNN): This machine learning architecture is designed to learn the internal structure of graph data and has great potential in molecular modeling, social network analysis, and recommendation systems [2]. They are based on graph algebra, spectral graph theory, and complex synthesis of differential calculus.

The mathematical foundations of optimization and graph algorithms are not only useful for theoretical research, but also for solving a wide range of real-world problems.

Modern algorithms, especially machine learning models, are deeply embedded in our daily lives, and issues of fairness and interpretability are becoming increasingly important. Mathematical approaches play an important role in solving these problems.

Algorithmic fairness: Algorithms can reflect systematic bias, which leads to unfair decisions for certain groups of people. Defining fairness from a mathematical perspective is a complex issue, and various fairness measures have been proposed:

- Demographic parity: This requires that the probability of a favorable outcome be the same for all groups. Mathematically, this is expressed as $P(Y'=1 \mid G=0) = P(Y'=1 \mid G=1)$, where Y' is the outcome and G is the protected attribute (e.g., gender, race).
- Equal Opportunity: Requires that the probability of a favorable outcome for different groups be the same (assuming a truly favorable outcome). That is, $P(Y'=1 \mid Y=1, G=0) = P(Y'=1 \mid Y=1, G=1)$.
- Equalized Odds: This condition also implies equal opportunities for both positive and negative classes: $P(Y'=1 \mid Y=y, G=0) = P(Y'=1 \mid Y=y, G=1)$ for all y.

These fairness measures are based on statistical theory and probability theory, and provide a mathematical framework for identifying and reducing algorithmic bias. Ensuring fairness often involves optimizing the objective function by adding fairness constraints.

Algorithm interpretability (XAI - eXplainable AI): Understanding how black box models (e.g., deep neural networks) make decisions is becoming increasingly important, especially in high-risk areas (e.g., medicine, law). Various mathematical methods are being developed to improve explainability:

- Local interpretability: Methods such as LIME (Local Interpretable Modelagnostic Explanations) are based on analyzing the changes in the output resulting from small changes in the input data to explain a specific part of the model . This method is based on linear models and simple statistical analysis.

- Global interpretability: SHAP (SHapley Additive exPlanations) values use the Shapley value concept derived from game theory. They allow for a fair distribution of how much each feature contributes to the model's prediction. SHAP values are based on combinatorics and game theory.
- Visualization methods: Methods such as saliency maps or grad-CAM rely on differential calculations to visually show which parts of the model are being focused on in convolutional neural networks.

These mathematical approaches help make algorithmic decisions more transparent, reliable, and fair, which is important for building trust in artificial intelligence in society.

Conclusion

This article has analyzed the complex and multifaceted nature of the mathematical foundations of new algorithmic solutions. We have demonstrated the importance of fundamental mathematical principles underlying algorithmic developments in areas ranging from machine learning algorithms to quantum computing, blockchain technology, optimization, and graph theory. Subjects such as linear algebra, differential and integral calculus, probability theory, statistics, information theory, cryptography, and discrete mathematics not only form the theoretical basis of these algorithms, but are also essential for their practical effectiveness and reliability.

The article also analyzes mathematical approaches to modern problems such as algorithmic fairness and interpretability. Statistical fairness measures and game theorybased explanation methods provide the necessary tools to mitigate the social impact of algorithms and increase their transparency.

Future developments in these areas require thorough mathematical research. Quantum computing, the protection of personal data in blockchain technology, and new hybrid models and algorithms for solving complex optimization problems, as well as formalizing the ethical and legal aspects of decision-making from a mathematical perspective. A deep understanding of mathematical concepts and their integration with innovative algorithmic design is crucial for fully realizing the potential of future technologies. Research in this area not only contributes to the development of science, but also helps to solve complex problems facing society.

References:

1. Goodfellow, I., Bengio, Y., & Courville, A. (2016). Deep Learning. MIT Press. – https://www.deeplearningbook.org/
2. Liquet, B., Moka, S., & Nazarathy, Y. (2024). Mathematical Engineering of Deep Learning. CRC Press. – https://www.crcpress.com/Mathematical-Engineeringof-Deep-Learning/Liquet-Moka-Nazarathy/p/book/9781032128713
3. Nielsen, M. A., & Chuang, I. L. (2010). Quantum Computation and Quantum Information. Cambridge University Press. –
4. https://www.cambridge.org/core/books/quantum-computation-and-quantuminformation/E8812604323E573390231908611C3D62

5. Narayanan, A., Clark, J., & Boneh, D. (2016). Bitcoin and Cryptocurrency Technologies: A Comprehensive Introduction. Princeton University Press. – https://www.cs.princeton.edu/~arvindn/bitcoinbook/
6. Boyd, S., & Vandenberghe, L. (2004). Convex Optimization. Cambridge University Press. – https://web.stanford.edu/~boyd/cvxbook/