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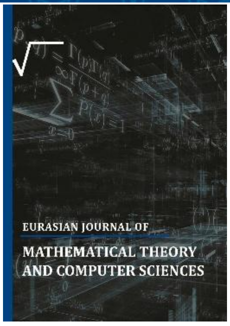


# LOCAL PATH-PRESERVING AUTOMORPHISMS IN THE ALGEBRA OF ASYMPTOTICALLY DIFFERENTIABLE FUNCTIONS

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https://doi.org/10.5281/zenodo.19450230

**ARTICLE INFO**

Received: 29th March 2026
Accepted: 05th April 2026
Online: 06th April 2026

**KEYWORDS**

*Asymptotic differential algebra, local automorphism, differential operator, idempotent, track maintenance.*

**ABSTRACT**

*This article investigates the structure of asymptotic differential algebras. The main attention is paid to the differential operator of local automorphisms preserving tracks and the property of commutativity. The study showed that if global automorphisms are commutative with the differential operator, then local automorphisms also retain this property. The problem of preserving the differential of complex functions is also considered.*

# ASIMPTOTIK DIFFERENSIALANUVCHI FUNKSIYALAR ALGEBRASIDA YO'LAKLARNI SAQLOVCHI LOKAL AVTOMORFIZMLAR

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https://doi.org/10.5281/zenodo.19450230

**ARTICLE INFO**

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*Asimptotik differensial algebra, lokal avtomorfizm, differensial operator, idempotent, yo'laklarni saqlash.*

**ABSTRACT**

*Ushbu maqolada asimptotik differensial algebralar strukturasi tadqiq qilinadi. Asosiy e'tibor yo'laklarni saqlovchi lokal avtomorfizmlarning differensial operator bilan kommutativlik xossasiga qaratilgan. Tadqiqot natijasida, agar global avtomorfizmlar differensial operator bilan kommutativ bo'lsa, lokal avtomorfizmlar ham ushbu xossani saqlashi isbotlangan. Shuningdek, murakkab funksiyalar differensialining saqlanishi masalasi ko'rib chiqilgan.*

## Kirish

Differensial algebralar va ularning avtomorfizmlari uzoq vaqtdan beri matematikaning muhim tadqiqot yo'nalishlaridan biri bo'lib kelgan. So'nggi yillarda kommutativ regular algebralar va ular bilan bog'liq operatorlar nazariyasida muhim yutuqlarga erishildi. Xususan, Ayupov, Kudaybergenov va Karimov tomonidan [1, 2] da kommutativ regular algebralarning izomorfizmlari va differensiallashlar fazosining o'lchami o'rganilgan.

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Shu bilan birga, asimptotik differensiallanuvchi funksiyalar algebrasi — differensial algebralarning umumlashmasi bo'lib, unda elementlar asimptotik qatorlarga yoyiladi va differensiallash operatori bu yoyilmaga mos ravishda ta'sir qiladi. Bunday algebralar [4, 3] da o'rganilgan bo'lib, ular matematik fizika va analizning turli sohalarida qo'llaniladi.

Ushbu maqolada biz asimptotik differensial algebralarda yo'laklarni saqlovchi lokal avtomorfizmlar tushunchasini kiritamiz va ularning muhim xossalaridan biri — murakkab funksiya differensialini saqlash xossasini isbotlaymiz. Bu natija [1] da olingan global avtomorfizmlar haqidagi natijalarni lokal holatga umumlashtiradi va asimptotik strukturalar nazariyasida yangi imkoniyatlar yaratadi.

## 2 Asosiy tushunchalar va ta'riflar

Ushbu bo'limda maqolada qo'llaniladigan asosiy tushunchalar va ta'riflar keltirilgan.

**Ta'rif 2.1** $A$ *to'plami* $\mathbb{F}$ *maydoni (xarakteristikasi 0) ustidagi* ***algebra*** *deyiladi, agar:*

1. $A$ vektor fazo bo'lsa;
2. $A$ da **ko'paytma** amali $\cdot : A\times A\to A$ aniqlangan bo'lib, quyidagi shartlarni qanoatlantirsa:

- Assotsiativlik: $\forall a,b,c\in A:(ab)c=a(bc)$;
- Kommutativlik: $\forall a,b\in A: ab=ba$;
- Chiziqlilik: $\forall a,b\in A,\lambda\in\mathbb{F}:\lambda(ab)=(\lambda a)b=a(\lambda b)$;
- Unital: $\exists 1\in A,\forall a\in A: a\cdot 1=a$.

**Ta'rif 2.2** $f\in AD^{(n)}(0,1)$ *funksiyasi* ***asimptotik differensiallanuvchi*** *deyiladi, agar har bir* $x_0\in(0,1)$ *nuqtada* $\{A_k\}_{k=0}^{n}\subset\mathbb{C}$ *sonlar mavjud bo'lib,*

$$f(x)=\sum_{k=0}^{n}A_k(x-x_0)^k+o((x-x_0)^n)$$

$x\to x_0$ da o'rinli bo'lsa. Bu yerdagi $A_k$ koeffitsiyentlari $f$ funksiyaning $x_0$ nuqtadagi $k$-tartibli **asimptotik hosilalari** deyiladi.

**Ta'rif 2.3** $AD^{(n)}(0,1)$ *orqali* $(0,1)$ *intervalda aniqlangan quyidagi xossaga ega bo'lgan funksiyalar algebrasini belgilaymiz:* $f\in AD^{(n)}(0,1)$ *agar har bir* $x_0\in(0,1)$ *nuqtada shunday* $A_0,A_1,\dots,A_n\in\mathbb{C}$ *sonlari mavjud bo'lib,*

$$f(x)=\sum_{k=0}^{n}A_k(x-x_0)^k+o((x-x_0)^n)$$

$x\to x_0$ da o'rinli bo'lsin. Bu yerdagi $A_k$ koeffitsiyentlari $f$ funksiyaning $x_0$ nuqtadagi $k$-tartibli asimptotik hosilalari deyiladi.

**Ta'rif 2.4** $(A,D)$ *juftligi* ***asimptotik differensiallanuvchi funsiyalar algebrasi*** *deyiladi, agar:*

1. $A$ kommutativ unital algebra bo'lsa;

2. $D: A \to A$ chiziqli operator bo'lib, quyidagi shartlarni qanoatlantirsa:

- Leybnits qoidasi: $\forall a,b \in A: D(ab) = D(a)b + aD(b)$;

- $A$ da $D$-asimptotik yoyilma mavjud: har bir $a \in A$ uchun $\{e_i\}_{i\in I}$ — o'zaro ortogonal idempotentlar to'plami va $\{\lambda_i\}_{i\in I} \subset \mathbb{F}$, $r(x) \in A$ mavjudki,

$$a = \sum_{i=1}^{n} \lambda_i e_i + r(x),$$

bu yerda $r(x)$ **asimptotik qoldiq had** deyiladi va $D^k r(x) \to 0$ $(k = 0,1,\ldots)$ asimptotik ma'noda;

- Idempotentlar differensiali nolga teng: $\forall e \in \nabla(A): De = 0$, bu yerda $\nabla(A)$ — $A$ dagi barcha idempotentlar to'plami.

**Ta'rif 2.5** $A$ *kommutativ regular algebra bo'lsin.* $a \in A$ *elementning (support) deb*

$$s(a) = \inf\{e \in \nabla(A): ea = a\}$$

idempotentiga aytiladi. $a$ va $b$ elementlar **ortogonal** deyiladi, agar $s(a)s(b) = 0$ bo'lsa.

**Ta'rif 2.6** $T: A \to A$ *chiziqli operatori **yo'laklarni saqlovchi** (band preserving) deyiladi, agar quyidagi ekvivalent shartlardan biri bajarilsa:*

1. $\forall a \in A: s(T(a)) \le s(a)$;

2. $\forall a,b \in A: s(a)s(b) = 0 s(T(a))s(b) = 0$;

3. $\forall a \in A, \forall e \in \nabla(A): ea = ae T(a) = T(a)$.

Bu xossa operatorning idempotentlar strukturasini saqlashini ifodalaydi.

**Ta'rif 2.7** $\Phi: A \to A$ *akslantirishi **avtomorfizm** deyiladi, agar:*

1. $\Phi$ biyektiv bo'lsa;

2. $\Phi(a+b) = \Phi(a) + \Phi(b)$ (additivlik);

3. $\Phi(ab) = \Phi(a)\Phi(b)$ (multiplikativlik);

4. $\Phi(\lambda a) = \lambda\Phi(a)$ (chiziqlilik);

5. $\Phi(s(a)) = s(\Phi(a))$ (tayanchni saqlash).

**Ta'rif 2.8** $L: A \to A$ *chiziqli akslantirishi **lokal avtomorfizm** deyiladi, agar har bir* $a \in A$ *uchun shunday* $\Phi_a \in Aut(A)$ *(global avtomorfizm) mavjud bo'lsaki,*

$$L(a) = \Phi_a(a).$$

Bu ta'rif lokal avtomorfizm har bir elementda global avtomorfizm kabi harakat qilishini, lekin turli elementlar uchun turli global avtomorfizmlar mos kelishi mumkinligini ifodalaydi.

**3.Asoaiy natija**

**Teorema 3.1** *Faraz qilaylik,* $(\mathcal{A}, D)$ *asimptotik differensialanuvchi funksiyalar algebrasi va* $T: \mathcal{A} \to \mathcal{A}$ *yo'laklarni saqlovchi (band preserving) lokal avtomorfizm bo'lsin. Agar* $T$ *ga mos keluvchi har bir* $\Phi_f$ *global avtomorfizmi* *differensial operator* $D$ *bilan kommutativ bo'lsa* ($\Phi_f \circ D = D \circ \Phi_f$), *u holda* $T$ *operatori ham* $D$ *bilan kommutativdir, ya'ni:*

$$T \circ D = D \circ T$$

Isbotni bir necha mantiqiy bosqichda amalga oshiramiz:

1-bosqich: Elementning strukturaviy yoyilmasi

Asimptotik differensial algebra ta'rifiga ko'ra, ixtiyoriy $f \in \mathcal{A}$ elementini o'zaro ortogonal idempotentlar $\{e_i\}_{i=1}^n$ va asimptotik koeffitsientlar $\{c_i\}_{i=1}^n$ orqali quyidagicha ifodalash mumkin:

$$f = \sum_{i=1}^{n} c_i e_i + r$$

bu yerda $r$ -- asimptotik qoldiq had.

2-bosqich: Idempotentlar ustidagi ta'sir.

$T$ operatori yo'laklarni saqlovchi bo'lganligi sababli, u idempotentlar strukturasini saqlaydi va har bir idempotent uchun $T(e_i) = e_i$ sharti bajariladi. Asimptotik differensial algebra xossasiga ko'ra, barcha idempotentlarning differensiali nolga teng, ya'ni $D(e_i) = 0$. Bundan quyidagi tenglik kelib chiqadi:

$$D(T(e_i)) = D(e_i) = 0 \quad \text{va} \quad T(D(e_i)) = T(0) = 0$$

Demak, operatorlar idempotentlar qismida kommutativdir.

3-bosqich: Lokal avtomorfizm va global kommutativlik.

$T$ lokal avtomorfizm bo'lgani uchun, har bir $c_i$ koeffitsienti uchun shunday $\Phi_i$ global avtomorfizmi mavjudki, $T(c_i) = \Phi_i(c_i)$ bajariladi. Teorema shartiga ko'ra, $\Phi_i \circ D = D \circ \Phi_i$.

Endi $T(D(f))$ ifodasini hisoblaymiz:

$$T(D(f)) = T\left(\sum_{i=1}^{n} D(c_i)e_i + D(r)\right) = \sum_{i=1}^{n} T(D(c_i))e_i + T(D(r))$$

Lokal xossaga ko'ra $T(D(c_i)) = \Phi_i(D(c_i))$. Global kommutativlikdan foydalansak:

$$T(D(c_i)) = \Phi_i(D(c_i)) = D(\Phi_i(c_i)) = D(T(c_i))$$

4-bosqich: Yakuniy xulosa.

Yuqoridagi natijalarni birlashtirib va asimptotik qoldiq had uchun $T(D(r)) = D(T(r))$ ekanligini hisobga olib, quyidagiga ega bo'lamiz:

$$T(D(f)) = \sum_{i=1}^{n} D(T(c_i))e_i + D(T(r)) = D\left(\sum_{i=1}^{n} T(c_i)e_i + T(r)\right) = D(T(f))$$

Shunday qilib, ixtiyoriy $f \in \mathcal{A}$ uchun $T(D(f)) = D(T(f))$ tengligi isbotlandi.

## References:

1. Ayupov, Sh. A., Kudaybergenov, K. K., Karimov, Kh. K. (2022). Isomorphisms of commutative regular algebras. *Positivity*, 26(11), 1-15.

2. Ayupov, Sh. A., Kudaybergenov, K. K., Karimov, Kh. K. (2023). Isomorphism between the algebra of measurable functions and its subalgebra of approximately differentiable functions. *Vladikavkaz Mathematical Journal*, 25(2), 25-37.

3. Ber, A. F., Kudaybergenov, K. K., Sukochev, F. A. (2022). Derivations of Murray-von Neumann algebras. *Journal für die Reine und Angewandte Mathematik*, 791(10), 283-301.

4. Bouzar, C., Slimani, M. (2023). Asymptotic automorphisms in differential algebras. *Vladikavkaz Mathematical Journal*, 25(2), 24-36.

5. Johnson, B. E. (1969). Local automorphisms of operator algebras. *Journal of Functional Analysis*, 3(2), 171-186.

6. Kadison, R. V., Liu, Z. (2014). A note on derivations of Murray-von Neumann algebras. *Proceedings of the National Academy of Sciences U.S.A.*, 111(6), 2087-2093.