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PROOF OF CAUCHY'S THEOREM IN GENERAL. CONCEPT OF HOMOTOPY PATH

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ABSTRACT

It is known that one of the important formulas for holomorphic functions is Cauchy's integral formula. Using this formula, the function given on the boundary can be restored holomorphically within the domain.

Now we turn to the general proof of Cauchy's theorem, which is the basis of the theory of integration of holomorphic functions.The confirmation of this theorem is as follows: if the integral path ends in the domain fixed or remains closed and continuously deformed, the integral of the holomorphic function along this path does not change.In order to fully formulate the theorem,first of all,we clarify what we mean by continuous deformation of the path.

For the sake of simplicity,we assume that the parameter t for all paths changes in one section $J=[0,1]$.This assumption does not contradict the generality,because the path of the integral can always be replaced by an equivalent path by performing a possible substitution, and the value of the integral will not change in this case (change of variable in the integral).

Suppose we are given paths $\gamma_0 : J \to D$ and $\gamma_1 : J \to D$ with common vertices: $\gamma(0)=\gamma_1(0)=a$, $\gamma_0(1)=\gamma_1(1)=b$

Definition. 1. In the field D, γ_0 and γ_1 are called homotopic paths if there is a continuous reflection $\gamma(s,t): J\times J \to D$ such that

$$\gamma(0,t)\equiv\gamma_0(t), \qquad \gamma_1(1,t)\equiv\gamma_1(t) \qquad (t\in J),$$
$$\gamma(s,t)\equiv a, \qquad \gamma(s,t)\equiv b \qquad (s\in J)$$

if the conditions are right.

It's like that γ_0 and γ_0 closed paths are D called homotopes in the field, if so continuous $\gamma(s,t): J\times J\to D$ if there is reflection,

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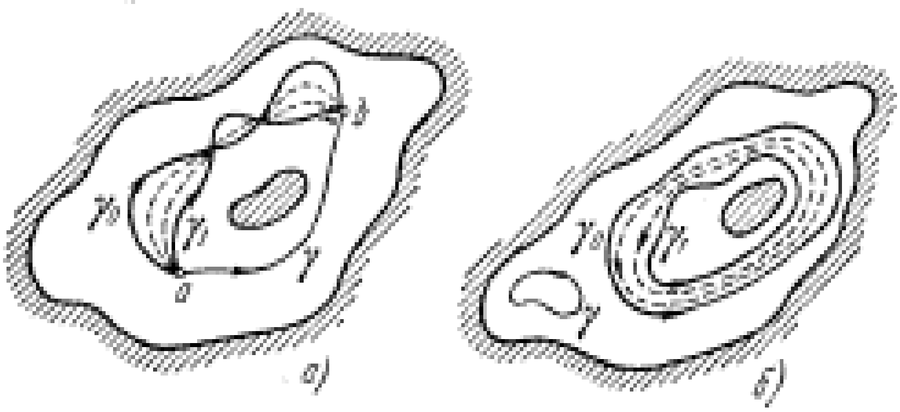
$$\gamma(s,t) \equiv \gamma_0(t), \qquad \gamma(1,t) \equiv \gamma_1(t) \qquad (t \in J),$$
$$\gamma(s,0) \equiv \gamma(s,t), \qquad\qquad (s \in J)$$

if the conditions are right.



If there are D homotopy paths γ_0 va γ_1 in the field $\gamma_0 \Delta \gamma_1$ in the form

to be determined.Note. Each appointed $s = s_0 \in J$ in $\gamma(s_0,t): J \to D$ the function defines a path in domain D.These paths change continuously as s_0 changes, and their class "connects" closed paths γ_0 and γ_1 in domain D (in the picture they are indicated by dots). Thus, the homotopy of two paths in D means that they can be continuously deformed into each other in D. In the figure, closed paths γ_0 and γ_1 are homotopic, and γ is not homotopic to them. If γ_0 and γ_1 are homotopes, we denote $\gamma_0 \Delta \gamma_1$ Obviously satisfies the axioms of homotopy equivalence reflexivity,symmetry and transitivity.

Therefore, it is possible to distinguish a class of paths that have common vertices in a given area, or are mutually homotopic to closed paths. Such classes are called homotopic classes.

In the class of closed paths, the class of paths homotopic to zero has a special place.

Definition .2. A closed path γ is called homotopy to zero in the domain D if there exists a continuous reflection $\gamma(s,t): J \times J \to D$ satisfying (2) such that $\gamma_1(t) \equiv const$ (that is, γ can be transformed into a point by continuous deformation in the domain D

An arbitrary closed path γ is homotopic to zero in a one-connected field D, which means that paths with arbitrary two common vertices in one-connected fields are mutually homotopic. With the help of this property, it is also possible to introduce the concept of a connected sphere.

Since the homotopy of two paths is preserved under all possible parametric permutations, this concept can also be applied to curves.

Two curves are said to be homotopic in D if the closed paths γ_0 and γ_1 forming the class of these curves are homotopic in D. In introducing the concept of integral, we used the path, that is, we defined the integral over the path. In the next steps, we saw that actually the integral is defined not by a path, but by a curve, which is a class of paths equivalent to this path. In general, Cauchy's theorem confirms that in the case of a holomorphic function, it is determined not by the integral curve, but by the class of homotopic paths to which this curve belongs. In other words, the following statement is appropriate.

Theorem .1. (Koshi). If $f \in H(D)$, two closed paths γ_0, and γ_1 have a common vertex in the domain d or are mutually homotopy as closed paths, then

$$\int_{\gamma_0} f\ dz = \int_{\gamma_1} f\ dz \quad (3)$$

will be.

This theorem gives rise to the classical form of Cauchy's theorem.

Theorem .2. If $f \in H(D)$, then the integral from the function f to zero on the closed path $\gamma : J \to D$ in the field D is equal to zero, if $\int_{\gamma} f\ dz = 0$, then $\int_{\gamma} f\ dz = 0$ **Proof.** Since $\gamma = 0$by condition, the path γ can be deformed into a closed path $\gamma_1$lying on some circle $U \subset D$. According to the local theorem about the existence of a prime function, the function f has a prime function F in U, and accordingly, the prime of f with respect to γ_1 is the function $F \circ \gamma_1$. On the other hand, since γ_1 is closed (since it is $\gamma_1(0) = \gamma_1(1) = a$), according to the Newton-Leibens formula.

$$\int_{\gamma_1} f\ dz = F(a) - F(b) = 0$$

will be. By Theorem 1, the integrals obtained from f with respect to γ_1 and γ are equal to:

$$\int_{\gamma_1} f\ dz = \int_{\gamma} f\ dz$$

and therefore $\int_{\gamma_1} f\ dz = 0$

Since any closed curve in one-connected fields is homotopy to zero, Cauchy's theorem looks simpler in such fields.

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