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FUNKSIYALARNI LAGRANJ INTERPOLYATSION FORMULASI YORDAMIDA APPROKSIMATSIYALASH MASALASINI DASTURLASH HAQIDA

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https://www.doi.org/10.5281/zenodo.7961820

ARTICLE INFO

Received: 14th May 2023
Accepted: 22th May 2023
Online: 23th May 2023

KEY WORDS

ABSTRACT

Tajribadan ma'lumki, funksiyalarni Lagranj interpolyatsion formulasi yordamida approksimatsiyalash bo`yicha mavzusini o`zlashtirishda talabalar ancha qiyinchiliklarga duch keladilar. Ushbu maqolada dastlab asosiy tushunchalarni keltirib, masala tanlab, uni Lagranj interpolyatsion formulasi yordamida yechib, unga tuzilgan dastur kodini keltiramiz va natijalarni solishtiramiz.

$f(x)$ funksiya $[a, b]$ oraliqda aniqlangan bo'lsin, ushbu oraliqda $f(x)$ va $\varphi(x)$ funksiyalarning yaqinligi ta'minlangan bo'lsin. Berilgan kesmada

$a \le x_0 < x_1 < x_2 < \ldots < x_n \le b$

shart bo'yicha nuqtalar to'plami, ya'ni tugunlarni tanlab olamiz. Bunda tugunlarning soni $F(x)=\varphi(x,\bar{c})=c_1\varphi_1(x)+c_2\varphi_2(x)+\ldots+c_n\varphi_n(x)=\sum_{k=1}^{n} c_k\varphi_k(x)$ tenglikdagi $\bar{c}$ parametrlar soniga teng. Bu tugunlarda $f(x)$ funksiyaning qiymatlari ma'lum, ya'ni $y_i = f(x_i)$, $i=\overline{0,n}.$

Interpolyatsiya masalasi $F(x)=\varphi(x,\bar{c})=c_1\varphi_1(x)+c_2\varphi_2(x)+\ldots+c_n\varphi_n(x)=\sum_{k=1}^{n} c_k\varphi_k(x)$ ga mos ko'phadlarni tanlashga olib kelinadi:

$$P(x)=c_0x^n+c_1x^{n-1}+\ldots+c_{n-1}x+c_n=\sum_{k=0}^{n} c_kx^{n-k},$$

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Bunda c_k koeffitsiyentlar haqiqiy sonlardan iborat bo‘lib, quyidagi qoida asosida topiladi:

$$\sum_{k=0}^{n} c_k x_i^{n-k} = f(x_i) = y_i, \qquad i = \overline{0,n}.$$

Ma’lumki, bunday ko‘phadga interpolyatsion ko‘phad deyiladi.

$\sum_{k=0}^{n} c_k x_i^{n-k} = f(x_i) = y_i$, $i = \overline{0,n}.$ shart qo‘llanadigan $P(x) = c_0 x^n + c_1 x^{n-1} + \ldots + c_{n-1} x + c_n = \sum_{k=0}^{n} c_k x^{n-k}$, amaliyotga global interpolyatsiya deyiladi.

Agar $P(x) = c_0 x^n + c_1 x^{n-1} + \ldots + c_{n-1} x + c_n = \sum_{k=0}^{n} c_k x^{n-k}$ ko‘phad $f(x)$ funksiyaning aniqlanish sohasi bo‘lgan $[a, b]$ kesmaning faqat alohida qismlari uchun, ya’ni $m < n$ bo‘lgan m ta interpolyatsion tugun uchun qurilsa, u holda interpolyatsiyani lokal interpolyatsiya deyiladi.

Lagranj interpolatsiya formulasi, berilgan n ta nuqta orasidagi funksiyaning approksimatsiyasini topish uchun ishlatiladi. Bu formulaning asosiy ideyasi, berilgan n ta nuqta orasidagi funksiyaning bir nechta polinomlarga bo'linishi va har bir polinomni o'zining x qiymatida hisoblashdir. Lagranj interpolatsiyasi formulasi quyidagicha ifodalash mumkin:

Umumiy ko‘rinishdagi interpolyatsiyada interpolyatsion ko‘phad x_T ning aniqlanish sohasida barcha intervallar uchun $P(x) = c_0 x^n + c_1 x^{n-1} + \ldots + c_{n-1} x + c_n = \sum_{k=0}^{n} c_k x^{n-k}$, ko‘rinishda izlanadi, ya’ni $[x_0, x_n]$ uchun: $\varphi(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$.

a_i koeffitsiyentlarni aniqlash uchun $\sum_{k=0}^{n} c_k x_i^{n-k} = f(x_i) = y_i$, $i = \overline{0,n}.$ tenglamalar sistemasi tuziladi:

$$\begin{cases} a_0 + a_1 x_0 + \ldots + a_n x_0^n = y_0; \\ a_0 + a_1 x_1 + \ldots + a_n x_1^n = y_1; \\ \qquad \ldots \\ a_0 + a_1 x_n + \ldots + a_n x_n^n = y_n. \end{cases}$$

Ma’lumki, agar $i \neq j$ lar uchun $x_i \neq x_j$ shart o‘rinli bo‘lsa, tenglamalar sistemasi yagona yechimga ega bo‘ladi. Keltirilgan tenglamalar sistemasini yechish uchun chiziqli algebraik tenglamalar sistemasini yechish usullaridan foydalansa ham bo`ladi. Tenglamalar sistemasini to‘g‘ridan to‘g‘ri yechib, $F(x)$ funksiyani $\varphi(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ ko‘rinishida olish ham mumkin, bunda bir nechta hisoblashlar bitta jadval bo‘yicha bajariladi.

$y = f(x_T)$ ni bir martalik hisoblash uchun $\bar{a}$ vektor parametrlarini topish shart bo‘lmagan boshqa algoritmlar mavjud. Interpolyatsion ko‘phadlar esa $\{x_i, y_i\}$, $i = \overline{0,n}$ jadval qiymatlari orqali yoziladi. Bular Lagranj va Nyuton interpolyatsion ko‘phadlaridir.

Quyidagi misolni Lagranj interpolyatsion formulasi yordamida yechamiz: Jadval bilan berilgan $y=f(x)$ funksiya qiymatini $x = 2.5$ bo‘lgan hol uchun hisoblaymiz:

i	0	1	2	3	4
x_i	1	2	3	4	5
y_i	2	4	1	7	3

Masalani quyidagicha yechamiz: $L_n(x) = \sum_{j=0}^{n} y_j \prod_{\substack{i=0 \\ i\neq j}}^{n} \frac{x - x_i}{x_j - x_i}$ ga asosan ishchi formulani yozib olamiz:

$$L(x) = \sum_{i=0}^{n} y_i \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}.$$

Bizning holda $n = 4$ gacha, shu sababli:

$$L(x) = \sum_{i=0}^{4} y_i \prod_{\substack{j=0 \\ j\neq i}}^{4} \frac{x - x_j}{x_i - x_j} =$$

$$= y_0 \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + y_1 \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} +$$

$$+ y_2 \frac{(x-x_1)(x-x_0)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} + y_3 \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} + y_4 \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} =$$

Berilgan jadval asosida, xususan, n=1 va x_T = 2.5 bo‘lgan hol uchun Lagranj ko‘phadini tuzamiz:

$$L(x)=\sum_{i=0}^{1} y_i \prod_{\substack{j=0 \\ j\neq i}}^{1} \frac{x - x_j}{x_i - x_j} = y_0 \frac{x - x_1}{x_0 - x_1} + y_1 \frac{(x - x_0)}{(x_1 - x_0)} =$$

$$= 2 * \frac{x-2}{1-2} + 4 * \frac{(x-1)}{(2-1)} = 6x - 8;$$

Bu esa chiziqli interpolyatsion formula bilan ustma-ust tushadi. Berilgan jadval asosida n=2 va x_T = 2.5 bo‘lgan hol uchun Lagranj ko‘phadini tuzamiz:

$$y \approx L(x) = \sum_{i=0}^{2} y_i \prod_{\substack{j=0 \\ i\neq j}}^{2} \frac{x - x_j}{x_i - x_j};$$

Qaralayotgan $[x_1, x_4]$ intervalda

$x_0 = 2;$ $x_1 = 3;$ $x_2 = 4;$ $x_3 = 5;$

$y_0 = 4;$ $y_1 = 1;$ $y_2 = 7;$ $y_3 = 3;$

qiymatlarni olamiz. U holda 2–tartibli Lagranj interpolyatsion ko‘phadi hosil bo‘ladi:

$$y \approx L(x) = 4 \cdot \frac{(x-3)(x-4)(x-5)}{(2-3)(2-4)(2-5)} + 1 \cdot \frac{(x-1)(x-5)(x-2)}{(3-2)(3-4)(3-5)} + 7 \cdot \frac{(x-2)(x-3)(x-5)}{(4-2)(4-3)(4-5)} +$$

$$+ 3 \cdot \frac{(x-2)(x-4)(x-3)}{(5-2)(5-3)(5-4)} = 0.278x^3 - 10.1x^2 - 49.39x - 63.2$$

$x = 2.5$ bo‘lganda $y \approx L(x) = 1.47656$

Shunday qilib, Lagranj interpolyatsiyasi formulasi yordamida funksiyaning approksimatsiyasini topish uchun, biz berilgan *n* ta nuqta orasidagi funksiya qiymatlaridan foydalanib, Lagranj polinomlarini hisoblaymiz va ularni funksiya qiymatlariga ko'paytirib, yig'indisini hisoblaymiz. Shu yig'indidan hosil bo'lgan qiymat esa approksimatsiya qilingan nuqtaga mos keladi.

Funksiyani Lagranj interpolyatsion formulasi yordamida tuzilgan dasturni keltiramiz:

```
//Yoqubov Shehroz C++ da Lagranj  interpolyatsion formulasi dasturi
#include <iostream>
#include <vector>
#include <cmath>
using namespace std;
double lagrange_interpolation(vector<double> x, vector<double> y, double xi) {
    double yi = 0;
    int n = x.size();
    for (int i = 0; i < n; i++) {
        double term = y[i];
        for (int j = 0; j < n; j++) {
            if (i != j) {
                term *= (xi - x[j]) / (x[i] - x[j]);
            }
        }
        yi += term;
    }
    return yi;
}
int main() {
    vector<double> x = {1, 2, 3, 4, 5};
    vector<double> y = {2, 4, 1, 7, 3};
    double xi = 2.5;
    double yi = lagrange_interpolation(x, y, xi);
    cout << "f(" << xi << ") = " << yi << endl;
    return 0;
}
```

Dastur natijasi quyudagicha boladi.

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```
#include <vector>
#include <cmath>
using namespace std;

double lagrange_interpolation(vector<double> x, vector<double> y, double xi) {
    double yi = 0;
    int n = x.size();

    for (int i = 0; i < n; i++) {
        double term = y[i];

        for (int j = 0; j < n; j++) {
            if (i != j) {
                term *= (xi - x[j]) / (x[i] - x[j]);
            }
        }

        yi += term;
    }

    return yi;
}

int main() {
    vector<double> x = {1, 2, 3, 4, 5};
    vector<double> y = {2, 4, 1, 7, 3};

    double xi = 2.5;
    double yi = lagrange_interpolation(x, y, xi);

    cout << "f(" << xi << ") = " << yi << endl;

    return 0;
}
```

```
f(2.5) = 1.47656
```

"*f(x)* ushbu kod biz taxmin qilmoqchi bo'lgan funksiyani belgilaydi. Bu kod, Lagranj interpolyasiya formulasini va test ma'lumotlarini (x va y) o'z ichiga olgan C++ dasturidir. Test ma'lumotlari orqali $f(2.5)$ qiymatini hisoblaydi.

Dastur Lagranj interpolyasiya formulasini, x va y koordinatalarining ro`yxatlarini olib, berilgan $x_i$ nuqtasidagi qiymatni hisoblayadi. Bu usul bir nechta nuqtalardan o`tkazilgan grafiklarni interpolatsiyalash uchun qo`llaniladi.

"x" "xs ""i" "interpolyate (x, xs, ys) ""f(x) ""x" "xs" "ys" "[a, b] ""h" "h_interp ""f(x)".

Dastur natijasi taxminan = 1.4765 ga teng, ya'ni hisoblangan natija bilan deyarli bir xil $y \approx L(x) = 1.47656$.

Ushbu maqolada keltirilgan Lagranj interpolyatsion formulasi yordamida yechilib, dasturi tuzilgan masaladan talabalar mavzuni o`zlashtirishda foydalanishlari mumkin, shuningdek, undan foydalanib mavzuga doir berilgan topshiriqlarni bajarishlari mumkin. O`ylaymizki, talabalar keltirilgan natijalardan foydalanib, oliy matematika fani masalalarini dasturlash bilan bog`lab o`rganadilar.

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