



ALGEBRAIK VA TRANSENDENT SONLAR HAQIDA

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KALIT SO`ZLAR

tenglama, transsendental,
algebraik, sonli, haqiqiy,
ratsional, Germit
integrali.

ANNOTATSIYA

Shu ma'noda, ratsional koeffitsientli algebraik tenglama tushunchasi algebraik transsendental son tushunchasining umumlashmasidir, chunki "e" soni algebraik son emas.

Biz ratsional koeffitsientli algebraik tenglama

$$a_0x^n + a_1x^{n-1} + \dots + a_n = 0 \quad (1)$$

$$a_0 \neq 0, a_i \in \mathbb{Q}, i=0,1,2,\dots,n$$

berilgan bo`lsin.

Ta`rif 1. (1) tenglamaning har qanday ildizi algebraik son deyiladi.

Ta`rif 2. Har qanday algebraik bo`lmagan son transendent son deyiladi.

Ta`rif 3. Agar (1) tenglamaning chap qismini ikkita ratsional koeffitsientli ko`phadlar ko`paytmasi shaklida yozish mumkin bo`lmasa, u xolda (1) tenglama keltirilmaydigan tenglama deyiladi.

Ta`rif 4. Keltirilmaydigan tenglamani tartibi, uni qanoatlantiruvchi a-algebraik sonning tartibi deyiladi.

Ta`rif 5. $a_0=1$ b`lganda (1) tenglamaning ildizi butun algebraik son deyiladi.

Ta`rif 6. Agar (1) tenglama keltirilmaydigan bo`lsa, uning irratsional ildizi, algebraik irratsionallik deyiladi ($n \geq 2$).

Ushbu tadqiqotda e sonining transendentligini va algebraik irratsionalligi haqidagi masalalarni qaraymiz.

Sonning transendentligini isbotlashda, deyarli barcha hollarda algebraik sonning ratsional kasrga yaqinlashmasligini ko`rsatishdan foydalaniladi. Biz e sonining



transendentligini boshqa usulda ,Ermit ko`phadlari yordamida isbotlashga urinib ko`ramiz.

Umumiylikga zarar yetkazmasdan , (1) tenglamada $a_i \in \mathbb{Z}$, $a_0 \neq 1$, $i=0,1,\dots,n$ deb olish mumkin .

Faraz qilaylik $a_i \in \mathbb{Z}$, $a_0 \neq 1$, $a_n \neq 0$, $i=0,1,\dots,n$ da $x=e$ (1) tenglamani qanoatlantirishi , ya`ni

$$a_0 e^n + a_1 e^{n-1} + \dots + a_{n-1} e + a_n = 0 \quad (2)$$

tenglik bajarilmasligini ko`rsatish lozim.

Buni isbotlashni quyidagi tartibda olib boramiz:

- e soni va uning darajalarini shu songa yaqin ratsional qiymati bilan almashtiramiz.

$$e = \frac{M_1 + \varepsilon_1}{M}, e^2 = \frac{M_2 + \varepsilon_2}{M}, \dots, e^n = \frac{M_n + \varepsilon_n}{M} \quad (3)$$

bu yerda M, M_1, M_2, \dots, M_n - butun son , $\frac{\varepsilon_1}{M}, \frac{\varepsilon_2}{M}, \dots, \frac{\varepsilon_n}{M}$ - yetarlicha kichik kasr son.

e va e ning darajalarini (2) ga qo`yib va har ikkala tarafini M ga ko`paytirib quyidagini hosil qilamiz .

$$(a_0 M^n + a_1 M^{n-1} + \dots + a_{n-1} M + a_n) + (a_0 \varepsilon_n + a_1 \varepsilon_{n-1} + \dots + a_{n-1} \varepsilon_1) = 0 \quad (4)$$

-(4) yig`indining birinchi qo`shiluvchisi no`ldan farqli butun son ekanligini ;

-(4) yig`indining ikkinchi qo`shiluvchisi $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ larning tanlanganligiga qarab yetarlicha kichik to`g`ri kasr ekanligini ko`rsatamiz , natijada (2) tenglik bajarilmasligi kelib chiqadi.

$$a_0 M^n + a_1 M^{n-1} + \dots + a_{n-1} M + a_n \neq 0 \quad (5)$$

yig`indini butun son ekanligi ravshan , uning nolga teng emasligini ko`rsatish uchun M_1, M_2, \dots, M_n solarning umumiy bo`luvchi p (p -tub son) bo`lsa , $a_n M$ ning p ga bo`linmasligini ko`rsatish yetarli , chunki (5) yig`indi p ga bo`linmaydi , demak (5) yig`indi nol emas . Yuqoridagi mulohazani ko`rsatish uchun Ermit integrallaridan foydalanamiz . M sonini

$$M = \int_0^\infty \frac{z^{p-1} \{(z-1)(z-2)\dots(z-n)\}^p e^{-z}}{(p-1)!} dz \quad (6)$$

Kabi tanlaymiz , bu yerda n -(1) tenglamaning darajasi , p -qandaydir tub son , biz uni keyinroq aniqlaymiz .

M_k va ε_k ($k=1,2,\dots,n$) larni aniqlash uchun $0 < z < +\infty$ oraliqni k ga bog`liq ravishda oraliqlarga bo`lib , M_k va ε_k ($k=1,2,\dots,n$) larni quyidagicha aniqlaymiz:

$$M_k = e^k \int_k^\infty \frac{z^{p-1} \{(z-1)(z-2)\dots(z-n)\}^p e^{-z}}{(p-1)!} dz \quad (6a)$$

$$\varepsilon_k = e^k \int_0^k \frac{z^{k-1} \{(z-1)(z-2)\dots(z-n)\}^p e^{-z}}{(p-1)!} dz \quad (6b)$$

Matematik analiz kursidan , ma`lumki gamma - funksiyalar nazariyasiga ko`ra

$$\Gamma(\rho) = \int_0^\infty z^{\rho-1} e^{-z} dz$$

da ρ butun son bo`lsa , $\Gamma(\rho) = (\rho - 1)!$ bo`lar edi .

Ko`rinib turibdiki , ρ - butun son bo`lsa , $\Gamma(\rho)$ funksiyaning qiymati juda tez o`sadi. Ermit integrallarini gamma-funksiya orqali ifodalaymiz. Ermit integrali ostidagi funksiyaning suratini z ning darajasi kamayish tartibida yozamiz

$$f(z) = \{(z-1)(z-2)\dots(z-n)\}^p = \{z^n - \dots + (-1)^n n!\}^p = z^{np} - \dots + (-1)^n (n!)^p$$
 u holda ,



$$\begin{aligned}
M &= \int_0^{\infty} z^{p-1} \frac{f(z)e^{-z}}{(p-1)!} dz \\
&= \frac{(-1)^n (n!)^p}{(p-1)!} \int_0^{\infty} z^{p-1} e^{-z} dz \\
&+ \sum_{k=p+1}^{p+pn} \frac{C_k}{(p-1)!} \int_0^{\infty} z^{k-1} e^{-z} dz
\end{aligned}$$

bu yerda C_k - o'zgarmas butun son.

$p \in \mathbb{N}$ va tub son ekanligidan , $\Gamma(\rho) = (\rho - 1)!$, unda

$$M = (-1)^n (n!)^p + \sum_{k=p+1}^{p+pn} C_k \frac{(k-1)!}{(p-1)!}$$

bu yerda ham M ni topish kabi , mulohazalar yuritib

$M_k = \frac{p! A_k}{(p-1)!} = p A_k$, $k=1, \dots, n$ ni hosil qilamiz , demak $M_k : p$ barcha $k=1, \dots, n$ larda.

Shunday qilib (5) yig'indi p ga bo'linmaydigan butun son ekan , demak u nol soni emas .

Endi (4) yig'indining ikkinchi qo'shiluvchisi bilan shug'ullanamiz.

$$a_0 \varepsilon_n + a_1 \varepsilon_{n-1} + \dots + a_{n-1} \varepsilon_1 \quad (8)$$

ni xosil qilamiz . Bundan $\frac{(k-1)!}{(p-1)!}$ - butun son bo'lib , p -son umumiy bo'luvchi bo'ladi.

Masalan , $k=p+1$ bo'lsa , $\frac{p!}{(p-1)!} = p$, $k = p + 2$ bo'lsa , $\frac{(p+1)!}{(p-1)!} = p(p + 1)$ va h.k. U holda

$$\begin{aligned}
M &= (-1)^n (n!)^p \\
&+ p \{ C_{p+1} + C_{p+2} (p + 1) \\
&+ C_{p+3} (p + 1)(p + 2) + \dots \}
\end{aligned}$$

Kabi bo'ladi . Agar $p > n$ bo'lsa , M soni p ga bo'linmaydi . Faramizga ko'ra $a_n \neq 0$, unda $p > n$ shunday tanlaymizki , $a_n - p$ ga bo'linmaydi , shunday qilib , $a_n M$ ham p ga bo'linmaydi .

Endi M_k , $k=1, \dots, n$ sonlarni tekshiramiz . (6a) formulalarda $\delta = z-k$ almashtirish qilamiz . u holda

$$M_k =$$

$$\int_0^{\infty} \frac{(\delta + k)^{p-1} \{ (\delta + k - 1)(\delta + k - 2) \dots (\delta + k - n) \} e^{-\delta}}{(p-1)!} d\delta$$

yig'indida

$$\varepsilon_k = \int_0^k \frac{z^{p-1} \{ (z-1) \dots (z-n) \}^p e^{-z+k}}{(p-1)!} dz$$

Kabi olamiz va $p > n$, $p > a_n$ deb hisoblaymiz ($k=1, \dots, n$) va ε_k larni yuqoridan baholaymiz .

Faraz qilaylik , G va g_k ($k=1, \dots, n$) sonlar mos ravishda $z(z-1)(z-2) \dots (z-n)$ va $(z-1)(z-2) \dots (z-n) e^{-z+k}$ funksiyalar uchun ($0 \leq z \leq n$ bo'lgan holda) yuqoridan baho bo'lsin , ya'ni $z \in [0; n]$ bo'lganda



$$\begin{cases} |z(z-1) \dots (z-n)| \leq G \\ |(z-1)(z-2) \dots (z-n)e^{-z+k}| \leq g_k \end{cases}$$

bo`lsin . U xolda

$$|\varepsilon_k| \leq \int_0^k \frac{G^{p-1}g_k}{(p-1)!} dz = \frac{G^{p-1}g_k k}{(p-1)!} \quad (9)$$

G , g_k va p ga bog`liq bo`lmaganligi sababli p ning yetarlicha katta qiymatlarida ε_k larni yetarlicha kichik qilish mumkin . (8) va (9) tengsizliklarga ko`ra (8) ni yuqoridan baholaymiz.

$$\begin{aligned} |a_0\varepsilon_n + a_1\varepsilon_{n-1} + \dots + a_{n-1}\varepsilon_1| &\leq |a_0| \cdot \\ |\varepsilon_n| + |a_1| \cdot |\varepsilon_{n-1}| + \dots + |a_{n-1}| \cdot |\varepsilon_1| &\leq \end{aligned}$$

$$\begin{aligned} (|a_0|ng_n + |a_1|(n-1)g_{n-1} + \dots + \\ |a_{n-1}|g_1) \frac{G^{p-1}}{(p-1)!} \leq A(n) \cdot \frac{G^{p-1}}{(p-1)!} \end{aligned}$$

p ning yetarlicha katta qiymatlarida $\frac{A(n)G^{p-1}}{(p-1)!}$ ni xoxlagancha kichik , hattoki , 1 dan kichik qilish mumkin . Shunday qilib , (4) tenglikning birinchi qo`shiluvchi 0 dan farqli butun son , ikkinchi qo`shiluvchisi 1 dan kichik kasr son qilib p ni tanlash mumkin emas . U xolda (4) tenglikning bajarilishi mumkin emas . Demak , e soni algebraik son emas .

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