PPSUTLSC-2024
PRACTICAL PROBLEMS AND SOLUTIONS TO THE USE OF THEORETICAL LAWS IN THE SCIENCES OF THE 21ST CENTURY
TASHKENT, 6-8 MAY 2024
www.in-academy.uz

ESTIMATION OF UNKNOWN PARAMETER OF WEIBULL DISTRIBUTION IN INCOMPLETE MODELS OF STATISTICS

Berdimuradov Mirkamol
PhD student of National University of Uzbekistan, University Street 4, Tashkent 100149, Uzbekistan
mirkamol1403@gmail.com, ORCID ID: 0009-0006-5004-5938
https://doi.org/10.5281/zenodo.13358608

Abstract: This paper will discuss the estimation results of Weibull distribution with type 1 right-censored data using numerical methods. These methods involve simulations employing the Maximum Likelihood Estimation technique, utilizing both the Quasi-Newton rule and the Nelder-Mead simplex algorithm. The simulation includes generating random sample data from distribution with sample n sizes of 500 and 1000. The parameters used for the initial guess are obtained from example data of patients with lung cancer, specifically $k=2,\ \lambda=3$. Based on the simulation results of the two estimation methods, it is evident that parameter estimation using the Quasi-Newton rule outperforms the Nelder-Mead simplex algorithm when in an uncensored state. However, the estimated results of the Nelder-Mead method show better estimated values compared to the Quasi-Newton rule after a fixed censoring time. [see, graphs and tables below].

Keywords: Weibull distribution, Quasi-Newton, Nelder-Mead algorithm, MLE, Right censoring..

1 INTRODUCTION

Usually, when estimating unknown parameters using the MLE method, it is necessary to calculate the integral and derivatives of the cumulative distribution function $F(x)$. However, in some cases, due to the complexity of integration and derivation by the analytical method, numerical calculation in computer programs becomes necessary. In this thesis, the unknown parameters of the continuous Weibull distribution are estimated using numerical methods with random samples.The Weibull distribution is widely used in various fields, including modeling the distribution phenomena of fatigue and the lifespan of many devices, such as bearings, shafts, and motors. Luís Andrade Ferreira FEUP - Faculdade de Engenharia da Universidade do Porto Department of Mechanical Engineering Rua Dr. Roberto Frias, 4200-465, Porto, Portugal and José Luís Silva ESTV – Escola Superior Tecnologia de Viseu Department of Mechanical Engineering and Industrial Management Campus Politécnico, 3504-510, Viseu, Portugal are created an article which is based on Expectation-Maximization algorithm to estimate unknown parameters of Weibull distribution with MLE method. In this article, we compare the methods of estimating unknown parameters of the Weibull distribution using the Nelder-Mead and Quasi-Newton rules with numerical solutions in Matlab, as opposed to the EM algorithm. [1,2,4].

2 RESEARCH METHODOLGY

The estimation process utilizes various statistical techniques, methods, and procedures to analyze data concerning a variable of interest, such as the time elapsed from a clearly defined starting point (e.g., equipment installation) to a specific event (e.g., equipment or component failure). This process aims to estimate the distribution parameters for modeling the system under examination. These parameters are the distribution characteristics that reflect the behavior of a particular population and are therefore fixed for a specific system. Hence, the estimation of system parameters is derived from the data collected from the population. Parametric analysis assumes that the data conforms to a specific distribution, such as the Weibull distribution.

3 ANALYSIS OF LITERATURE ON THE SUBJECT

In probability theory and statistics, the Weibull distribution is a continuous probability distribution. It models a broad range of random variables, largely in the nature of a time to failure or time between events. The probability density function of a Weibull random variable is

$$f(x;\lambda,k)=\begin{cases}\dfrac{k}{\lambda}\left(\dfrac{x}{\lambda}\right)^{k-1}e^{-(x/\lambda)^k}, & x\ge 0\\ 0, & x<0\end{cases}$$

where $k>0$ is the shape parameter and $\lambda>0$ is the scale parameter of the distribution. Its complementary cumulative distribution function is a stretched exponential function. The Weibull distribution is related to a number of other probability distributions; in particular, it interpolates between the exponential distribution ($k=1$).

The cumulative distribution function for the Weibull distribution is

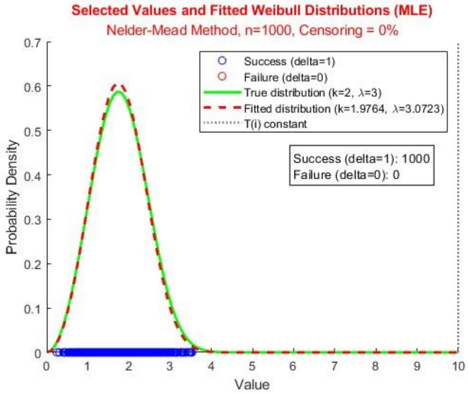
$$F(x;k,\lambda)=1-e^{-(x/\lambda)^k}$$

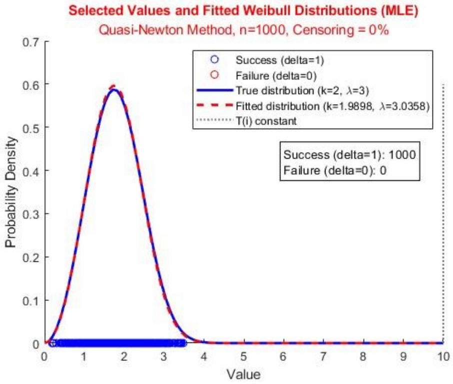
for $x \geq 0$, and $F(x;k,\lambda)=0$ for $x<0$.

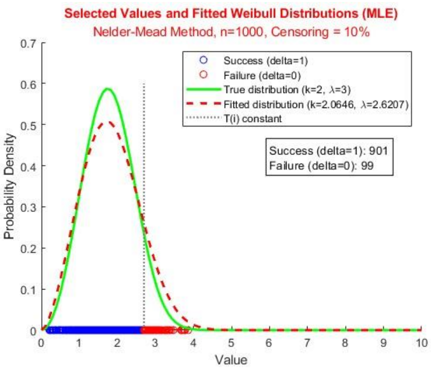
4 ANALYSIS AND RESULTS

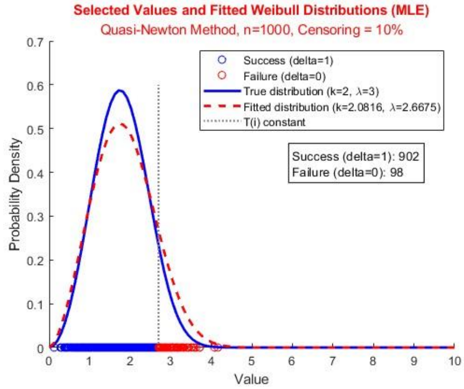
As we mentioned above, it is necessary to calculate the integral and derivatives of the cumulative distribution function $F(x)$. However, in some cases, due to the complexity of integration and derivation by the analytical method, numerical calculation in computer programs becomes necessary. To present some results obtained through numerical optimization using MATLAB for the Weibull distribution, below graphs and table 1 display the findings.

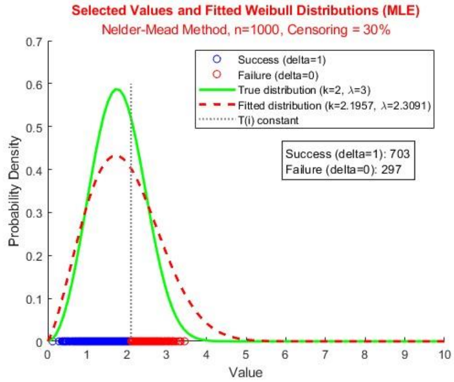
[Here, we can compare the results of both methods at different censoring levels.]

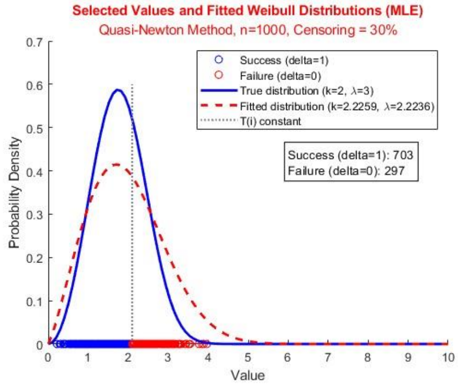












5 CONCLUSIONS AND SUGGESTIONS

Based on the simulation results of the two estimation methods, it is evident that parameter estimation using the Quasi-Newton rule outperforms the Nelder-Mead simplex algorithm when in an uncensored state. However, the estimated results of the Nelder-Mead method show better estimated values compared to the Quasi-Newton rule after a fixed censoring time.

Estimated of unknown parameters of Weibull distribution with two rules in MLE

Table 1

Interval [0;10]		
T	p%	Weibull Distribution
		Initial Guess k =2; λ=3

		$T_i \in [0;10]$ fixed constant	
		MLE by Quasi-Newton rule	MLE by Nelder-Mead rule
n=500			
10	0%	k =1.9815; λ =3.0215;	k =1.9118; λ =3.0412;
2.7	10%	k =2.0913; λ =2.705;	k =2.0742; λ =2.809;
2.2	30%	k =2.323; λ =2.2342;	k =2.1821; λ =2.401;
n=1000			
10	0%	k =1.9898; λ =3.0358;	k =1.9764; λ =3.0723;
2.7	10%	k =2.0816; λ =2.6103;	k =2.0646; λ =2.6207;
2.2	30%	k =2.2259; λ =2.2236;	k =2.1957; λ =2.3091;

REFERENCES

[1] Abernethy R B. The New Weibull handbook. Florida: Robert B. Abernethy, 2006.

[2] Akram M, Hayat A. Comparison of estimators of the Weibull distribution. Journal of Statistical Theory and Practice 2014; 8(2): 238-259, https://doi.org/10.1080/15598608.2014.847771.

[3] Balakrishnan N, Mitra D. Left truncated and right censored Weibull data and likelihood inference with an illustration. Computational Statistics and Data Analysis 2012; 56: 4011-4025, https://doi.org/10.1016/j.csda.2012.05.004.

[4] Balakrishnan N, Kundu D, Ng H K T. Point and interval estimation for a simple step-stress model with type-II censoring. Journal of Quality Technology 2007; 39: 35-47.

[5] Chambers R L, Steel D G, Wang S, Welsh A H. Maximum likelihood estimation for sample surveys. Boca Raton, Florida: Chapman and Hall/ CRC Press, 2012.

[6] Dempster A P, Laird N M, Rubin D B. Maximum likelihood from incomplete data via the EM algorithm. Journal of the Royal Statistical Society 1977; 39(1): 1-38.

[7] Efron B, Tibshirani R J. An introduction to the bootstrap. London: Chapman & Hall, 1993, https://doi.org/10.1007/978-1-4899-4541-9.

[8] Fang L Y, Arasan J, Midi H, Bakar M R A. Jackknife and bootstrap inferential procedures for censored survival data. AIP Conference Proceedings 2015; 1682: 1-6, https://doi.org/10.1063/1.4934631.

[9] Gijbels I. Censored data. Wires Computational Statistics 2010; 2: 178 - 188, https://doi.org/10.1002/wics.80.

[10] Guure C B, Ibrahim N A. Methods for estimating the 2-parameter Weibull distribution with type-I censored data. Journal of Applied Sciences, Engineering and Technology 2013; 5(3): 689-694.

[11] Karlis D, Xekalaki E. Choosing initial values for the EM algorithm for finite mixtures. Computational Statistics and Data Analysis 2003; 41: 577-590, https://doi.org/10.1016/S0167-9473(02)00177-9.

[12] Kinaci I, Akdogan Y, Kus C, Ng H K T. Statistical inference for Weibull distribution based on a modified progressive type-II censoring scheme. Sri Lankan Journal of Applied Statistics 2014; 1: 95-116, https://doi.org/10.4038/sljastats.v5i4.7786.

[13] Lawless J F. Statistical models and methods for lifetime data. New Jersey: John Wiley & Sons, 2003.

[14] McCool J I. Using the Weibull distribution, reliability, modeling and inference. New York: John Wiley & Sons, 2012, https://doi. org/10.1002/9781118351994.

[15] McLachlan G J, Krishnan T. The EM algorithm and extensions. New Jersey: John Wiley & Sons, 2008, https://doi. org/10.1002/97804701916