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MODELING THE KINEMATIC DISPLACEMENTS OF A TORSION SPRING FOR A HIGH-SPEED ELECTRIC TRAIN

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ABSTRACT

The article presents calculation schemes for modeling the kinematic movements of a torsion spring of a high-speed electric train using a graphical method..

In the world, when creating the chassis of high-speed rolling stock, it is necessary to re-set and solve the tasks of ensuring safety, smoothness, reducing vibrations at high speeds, therefore, the creation of new spring suspension designs using torsion springs is one of the most important tasks for railway transport.

Recently, torsion springs have been used in modern high-speed electric trains in spring suspension, since they have increased strength and reliability and are promising designs for high-speed electric rolling stock [1,2,3,4].

Torsion bar suspension has a number of advantages compared to leaf springs: torsion bars, with equal energy intensity, have less weight; advantage of torsion bars over coil springs lies in better layout capabilities of suspension of railway vehicles [5,6].

The strongest torsion bar is made of hardened, shot-peened, and pre-compressed steel, with permissible maximum stresses $\boldsymbol{\tau}$ [MPa], determined based on the experimental studies presented in monograph [4]:

$[\tau] = 1000 \div 1050$ ***MPa***, and the modulus of elasticity G = **7,4 • 10^4 MPa**.

The most common steel grades used for the spring suspension of electric trains are 50KhGA and 60S2Kh, as specified by the state standards GOST 14959-79. These grades are also utilized in the manufacturing of springs and leaf springs.

One of the most important aspects of calculating a torsion bar suspension is the correct preparation of a calculation scheme when modeling the kinematic movements of a torsion bar spring during the movement of a high-speed electric train, this is especially important on curves.

The determination of the torsion bar suspension stiffness in an electric locomotive with spring suspension is currently carried out using known formulas, which, however, are suitable only for two design forms of the guide device (single-lever suspension and double-lever, made in the form of a parallelogram) [4].

The use of these formulas to determine the stiffness of a double-lever trapezoidal torsion bar suspension, which has unequal lengths of the upper and lower levers of the guide device, can lead to significant errors. Below is a graphical-analytical method that allows one to determine the stiffness of the torsion bar suspension for any design of the guide device. A special experimental study confirmed the high accuracy of the proposed calculation method. It is known that the stiffness of a lever torsion bar suspension in the general case can be determined from the expression $C = T \cdot \frac{d^2}{ds^2} + C_T \cdot \left(\frac{d\theta}{ds}\right)^2$, (1)

where θ is the torsion angle; $C_T = \frac{dT}{dQ}$ is the torsion stiffness; s is the displacement of the torsion bar suspension during locomotive motion; T is the torsion moment.

With a linear characteristic, which is the case for most torsion bar designs, the torsion moment can be determined from the expression: $T = C_T \cdot \theta$. Using expression (1), we can obtain calculation formulas for determining the stiffness of a single-lever torsion bar suspension in the form proposed in [4]. Figure 1 shows a calculation scheme for determining the rigidity of a single-lever torsion bar suspension of a high-speed electric train.

Considering, according to Figure 1, that

$$\frac{d\theta}{ds} = \frac{1}{r \cdot \sin\varphi} = \frac{1}{\sqrt{r^2 - x^2}} , \quad (2)$$

$$\frac{d^2\theta}{ds^2} = \frac{-ctg\varphi}{r^2 \cdot \sin^2\varphi} = \frac{x}{\sqrt{(r^2 - x^2)^3}} , \quad (3)$$

we obtain two equations that differ only in form for determining the rigidity of a single-lever torsion bar suspension of an electric locomotive

$$C_P = \frac{G \cdot I_P}{L} \cdot \frac{1 - (\varphi - \varphi_0) \cdot ctg\varphi}{r^2 \cdot (\sin\varphi)^2} . \quad (4)$$

A similar expression for a particular solution of the problem was obtained in [5]

$$C_P = \frac{R_Z \cdot L \cdot x + G \cdot I_P}{(r^2 - x^2) \cdot L} . \quad (5)$$

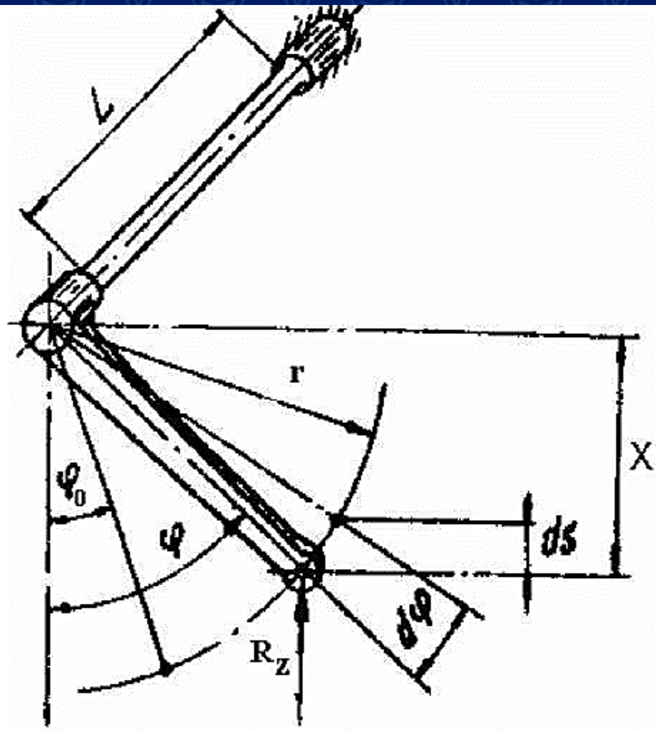

Figure 1. Calculation scheme for determining the rigidity of a single-lever torsion bar suspension of a high-speed electric train.

When using expression (5), it should be kept in mind that the magnitude of the displacement ***x*** must be substituted into the formula, taking into account its sign (the plus sign when moving upward from the static position). The magnitude of the load acting on the spring suspension in the area of the wheel pair (bogie) should be calculated for each position of the lever, determined by the angle $\boldsymbol{\varphi}$ or displacement ***x***, using the formula

$$\boldsymbol{R_z} = \frac{G \cdot I_P \cdot \theta}{L \cdot r \cdot \sin\varphi} = \frac{G \cdot I_P \cdot \theta}{L \cdot \sqrt{r^2 - x^2}} \quad . \tag{6}$$

In principle, it is possible to use expression (5) to derive calculation formulas for a double-wishbone trapezoidal torsion bar suspension. However, analytical calculation of the stiffness in this case would be quite labor-intensive. It is more practical to use a graphical-analytical method to determine the stiffness of a torsion bar suspension. We will explain the essence of this method for the suspension shown in Figure 2. A characteristic feature of this design is that the torsion bar is connected to the upper control arm.

Considering the equilibrium of the spring suspension of an electric locomotive, we can establish that three forces act on it in the plane of the drawing:

1. Q_z is the reaction from the railway track, applied at the center of the wheel-rail contact area and directed vertically;

2. Qy is the reaction from the lower suspension arm, the direction of which coincides with the axis of the arm, since the arm has hinges at both ends (at points D and E);

3. Q is the reaction from the lower arm, passing through the point of intersection of the lines of action of the forces Q_Z and Q_Y (point *0* in Fig. 2) into hinge B of the upper arm. If the magnitude of the vertical reaction Q_Z is known, then, knowing the direction of the

forces Q_Y and Q, their magnitudes can be easily found by graphically plotting a force triangle.

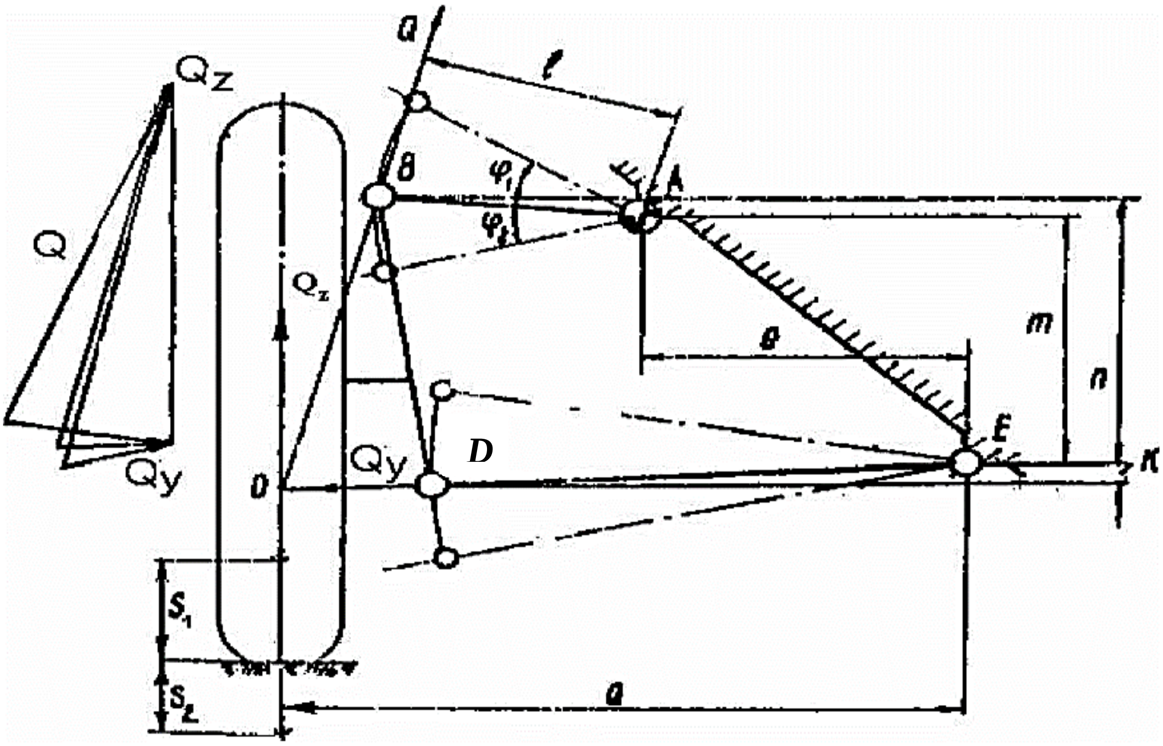

Figure 3. Calculation scheme for determining the rigidity of a double-wishbone torsion bar suspension of a high-speed electric train.

Such a construction can be made both for the static position of the suspension and for the dynamic position determined by the compression stroke of the torsion suspension when the electric locomotive is moving - downward movement of the suspension - S_1 or upward movement of the suspension - S_2.

Based on the equality of the work of force Q_Z and moment T, we can write, neglecting friction in the suspension, that

$$Q_z \cdot ds = T \cdot d\theta \; . \qquad (7)$$

From expression (7) we obtain $\frac{d\theta}{ds} = \frac{Q_z}{T}$. (8)

The magnitude of the torque twisting the torsion bar can be determined from the equation $T = Q \cdot \boldsymbol{l}$, (9)

where $\boldsymbol{l}$ is the arm of the force Q relative to the axis of rotation of the upper suspension arm (point A in Figure 2). Substituting the value of the moment T from expression (9) into expression (8), we obtain

$$\frac{d\theta}{ds} = \frac{Q_z}{Q \cdot l}. \qquad (10)$$

Thus, to determine the value of $\frac{d\theta}{ds}$, it is sufficient to determine the magnitude of the lever arm $\boldsymbol{l}$ and the ratio $\frac{Q_z}{Q}$. This must be done for a number of positions of the electric locomotive's torsion bar suspension. Since in this case it is important to know only the ratio $\frac{Q_z}{Q}$, and not their absolute values, it is advisable to construct force triangles for a number of positions during the movement of the electric locomotive's torsion bar suspension, assuming a constant value of the force Q_Z . To determine the $\frac{d\theta^2}{ds^2}$ value, a graph of the dependence of $\frac{d\theta}{ds}$ on the wheel travel in the electric locomotive's wheelset should be plotted, and then this curve should be graphically differentiated.

When substituting the value of $\frac{d\theta^2}{ds^2}$ into expression (10), the sign must be taken into account. The magnitude of the torsion bar torque can be determined from the expression

$$T = T_0 \pm C_T \cdot \boldsymbol{\theta} , \quad (11)$$

Where T_0 is the moment exerted on the torsion bar when the wheel in the electric locomotive's wheelset is in a static position; $\boldsymbol{\theta}$ is the torsion bar's twist angle (according to Figure 2, $\boldsymbol{\theta_1}$ is during the compression stroke, $\boldsymbol{\theta_2}$ is during the release stroke).

The value of T can be determined from the expression $T_0 = Q_o \cdot \boldsymbol{l_0}$, (12),

where $\boldsymbol{\theta_0}$ and $\boldsymbol{l_0}$ are, respectively, the force acting on the upper lever and the force arm under a static load on the wheel in the electric locomotive's wheelset.

The stiffness of an electric locomotive's torsion bar suspension and the nature of its variation depend greatly on the torsion bar's installation location, i.e., whether it is connected to the lower or upper control arm, the design of the guide device, and primarily the inclination angles of the upper and lower control arms.

Torsion bar suspensions are generally considered to have progressive characteristics, meaning their stiffness increases with increasing deflection. However, in reality, in most cases, torsion bar suspensions with trapezoidal guide device designs exhibit either very little progression or even regression, meaning their stiffness decreases with increasing deformation. The latter applies primarily to suspensions in which the torsion bar is connected to the lower control arm.

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