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EKSTREMAL MASALALARNI YECHISHDA TENGSIZLIKLAR USULIDAN FOYDALANISH.

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ABSTRACT
Ushbu maqolada Ekstremal masalalarni yechishda tengsizliklar usulidan foydalanish hususida alohida to'xtalib o'tilgan. Shuningdek, maqolada tengsizliklar haqida ham ayrim misollar keltirilgan.

Kirish. Tengsizliklar usuli va uning afzallliklari haqida. Ekstremal masalalarni yechishning turli usullari mavjud. Bunga avvalo, hosilalardan foydalanib yechish usulini (hosila usulini) misol sifatida keltirish mumkin. Qolaversa, geometrik usul xamda tengsizliklar usuli ham shular jumlasidandir. Biz bu maqolada ba'zi muhim klassik tengsizliklarning ekstremal masalalar yechishga tatbig'i (tengsizliklar usuli) bilan shug'ullanamiz. Ekstremal masalalarni yechishda tengsizliklar usulining mohiyati quyidagicha bayon etlishi mumkin.

Masalaning qo'yilishi. Ushbu maqolada quyidagi masalani tadqiq etamiz. Ixtiyoriy musbat $x_1, x_2, ..., x_K$ sonlar berilgan bo'lsin. Ular yordamida ushbu

$$A_k = \frac{x_1 + x_2 + ... + x_k}{k},$$

$$\Gamma_k = \sqrt[k]{x_1 x_2 \cdots x_k}$$

kattaliklarni tuzamiz. Bunda $A_k -$ berilgan sonlarning o'rta arifmetigi, Γ_k esa ularning $k-$tartibli o'rta geometrigi deyiladi. Mazkur kattaliklar orasida

$$\Gamma_\kappa \le A_\kappa \qquad (1)$$

munosabat mavjud bulib, unda tenglik belgisiga $x_1 = x_2 = ... = x_k$ bo'lganda va faqat shu holdagina erishiladi.

Masalaning yechilishi Ushbu $x_1 + x_2 + ... + x_k$ kattalik o'zgarmas bo'lsin, uni a bilan belgilaylik. U holda (1) tengsizlikdan ko'rinadiki, Γ_k ning eng katta qiymati faqat $A_\kappa = \frac{a}{\kappa}$ dan iborat bo'lishi mumkin. Bunga esa, $x_1 = x_2 = ... = x_k$ bo'lgandagina erishiladi. Demak, k ta musbat son yig'indisi o'zgarmas bo'lsa, shu sonlar ko'paytmasidan chiqarilgan $k-$ darajali ildiz o'zining eng katta qiymatiga ular o'zaro teng bo'lgandagina erishadi va bu eng katta qiymat $A_к = \frac{a}{к}$ ning o‘zi bo‘ladi. Shunday qilib, $\max Г_k = \frac{a}{к}$.

2) Endi $x_1 x_2 \ldots x_k$ kattalik o‘zgarmas son deylik, uni b bilan belgilaymiz. U holda (1) tengsizlikdan ravshanki, A_k ning eng kichik qiymati $\sqrt[k]{b}$ bo‘lishi mumkin holos. Bunga esa $x_1 = x_2 = \ldots = x_k$ bo‘lgandagina erishiladi. Demak k ta musbat son ko‘paytmasi o‘zgarmas bo‘lsa, shu sonlarning o‘rta arifmetigi o‘zining eng kichik qiymatiga ular o‘zaro teng bo‘lgandagina erishadi va bu eng kichik qiymat $Г_k = \sqrt[k]{b}$ ning o‘zi bo‘ladi. Shunday qilib, $\min A_k = \sqrt[k]{b}$.

Tengsizliklar usuli ma’lum sinf ekstremal masalalarni yechishda hosila usuliga qaraganda ba’zi afzalliklarga ega:

1) ekstremal qiymati topilishi kerak bo‘lgan funksiyaning hosilasi hisoblanmaydi;

2) Topilishi kerak bo‘lgan ekstremal qiymat (ekstremal hajm, yuza, uzunlik, burchak, vaqt va hokazolar) darhol topiladi;

3) Shu ekstremal qiymatga erishiladigan nuqta (hosila usulidagi kritik nuqtalar ichida funksiyaga ekstremal qiymat beradigani) ham darhol topiladi;

4) Tengsizliklar usuli ekstremal qiymati topilishi lozim bo‘lgan funksiya ko‘p argumentli bo‘lganda ham qo‘llanilishi mumkin.

2. Tengsizliklardan geometriya kursi va ekstremal masalalarni yechishda foydalanish. Endi (1) tengsizlikni avval planimetriyadagi, so‘ngra stereometriyadagi ba’zi ekstremal masalalarni yechishga tatbiq etamiz. Bu maqolada planimetriyaga oid quyidagi masala ko‘ramiz.

Masalaning amaliy tadbiqlari. Asosi c va balandligi h bo‘lgan uchburchakka bir tomoni asosda, ikki uchi uchburchakning yon tomonlarida bo‘lgan eng katta yuzali to‘g‘ri to‘rtburchak chizilsin.

Yechish: Chizmada ΔABC va □ $A_1, B_1, B_2 A$ lar chizilgan. Unda $AB = c,\ CD = h$ deylik. Ushbu $A_1B_1 = A_2B_2 = x$, $A_1A_2 = ED = B_1B_2 = y$ belgilashlarni kiritamiz. ΔABC va □ A_2, B_2, C, D larning o‘hshashligidan $\frac{c}{h} = \frac{x}{h-y}$, bundan $x = \frac{c}{h}(h-y)$.

$A_1B_1B_2A_2$ to‘g‘ri to‘rtburchakning yuzi $S(y) = xy = \frac{c}{h} y(h-y)$, ammo $y(h-y) \le \left(\frac{h}{2}\right)^2$ va bu tengsizlikda tenglik «=» belgisi $y = h - y$ bo‘lganda erishiladi. U holda $y = \frac{h}{2}$, $x = \frac{c}{2}$.

Demak, $S_{\max} = \frac{c}{h} \cdot \frac{h^2}{4} = \frac{1}{2} \cdot c \cdot \frac{h}{2} = \frac{1}{2} S_{\Delta ABC}$. Shunday qilib, to‘g‘ri to‘rtburchakning A_2B_2

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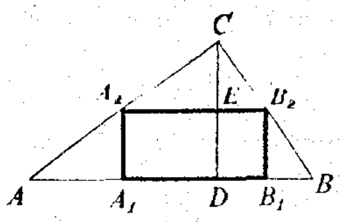
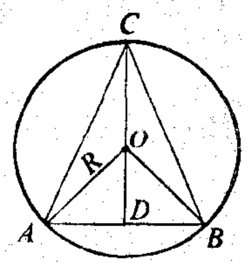
tomoni uchburchak balandligining o‘rtasidan uchburchak asosiga paralel qilib o‘tkazilsa, ya’ni A_2B_2 chiziq ΔABC ning o‘rta chizig‘i bo‘lsa, to‘g‘ri to‘rtburchakning yuzi eng katta bo‘ladi.

2-masala. Radiusi R bo‘lgan aylana ichiga chizilgan teng yonli uchburchaklar ichida yuzi eng kattasi topilsin.





Yechish. Chizmada, $AC + CB$, $OA = OB, CD = x$. Ravshanki, $DO = x - R, AB = 2AD = 2\sqrt{R^2 - DO^2} = 2\sqrt{R^2 - (x-R)^2} = 2\sqrt{2Rx - x^2}$.

Endi $S_{\Delta ABC}$ yuzini topamiz. $S_{\Delta ABC} = S(x) = \frac{1}{2} \cdot x \cdot \sqrt{2Rx - x^2} = \sqrt{x^3}\sqrt{2R - x}$.

Unda $0 < x < 2R$. Qiymatini topish uchun tengliklar usulini to‘g‘ridan to‘g‘ri qo‘llab bo‘lmaydi. $S(x)$ ni kvadratga ko‘taramiz: $S^2(x) = x^3(2R - x)$

va bu funksiyani quyidagicha yozib olamiz: $S^2(x) = \frac{1}{3} \cdot x \cdot x \cdot x \cdot (6R - 3x)$

Agar $x_1 = x, x_2 = x, x_3 = x, x_4 = 6R - 3x$ deb belgilasak $x_1 + x_2 + x_3 + x_4 = 6R = const$

va $x = 6R - 3x$, $x = \frac{3}{2}R$ kelib chiqadi. Demak, tengsizliklar usulini qo‘llash mumkin. $\Gamma_4 \le A_4$ tengsizlikka ko‘ra

$$S^2(x) = \frac{1}{3} \cdot x \cdot x \cdot x \cdot (6R - 3x) \le \frac{1}{3}\left(\frac{x + x + x + 6R - 3x}{4}\right)^4 = \frac{1}{3}\left(\frac{6R}{4}\right)^4 = \frac{1}{3}\left(\frac{3}{2}R\right)^4$$

Bundan $S(x) \le \frac{1}{\sqrt{3}}\left(\frac{3}{2}R\right)^2 = \frac{9}{4\sqrt{3}}R^2$, ya’ni $S_{\max} = S\left(\frac{3}{2}R\right) = \frac{9}{4\sqrt{3}}R^2$

kelib chiqadi. Endi ΔABC ning tomonlarini topib qo‘yaylik:

$$AB = 2\sqrt{2R \cdot \frac{3}{2}R - \left(\frac{3}{2}R\right)^2} = 2\sqrt{3R^2 - \frac{9}{4}R^2} - \sqrt{3}R,$$

$$AC - CB - \sqrt{AD^2 + DC^2} = \sqrt{\left(\frac{AB}{2}\right)^2 + x^2} = \sqrt{\frac{3}{4}R^2 + \frac{9}{4}R^2} = \sqrt{3}R$$

Shunday qilib, $AB = BC = CA = \sqrt{3}R$. Bu natija ΔABC ning teng tomonli ekanligi anglatadi. Shunday qilib, radiusi R bo‘lgan aylana ichiga chizilgan teng yonli uchburchaklar ichida yuzi eng kattasi teng tomonli uchburchak bo‘lar ekan.

Xulosa. Ushbu maqolada ko‘rilgan masalalar matematikaning eng muhim masalalaridan biri hisoblanadi. Maqoladan mustaqil o‘rganish va izlanishlar olib borish maqsadida akademik litsey, kasb-hunar kollejlari va oliy ta’lim muassasasi talabalari manba sifatida foydalanishlari mumkin.

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