



NUMERICAL STUDIES OF THE OSCILLATION PROCESS OF THE MOTOR-CARRIAGE OF THE AFROSIAB HIGH-SPEED ELECTRIC TRAIN

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ABSTRACT

The article presents the results of numerical studies of the process of oscillations of the motor-carriage of the high-speed electric train AFROSIAB in the horizontal plane during its movement in curves in the MATHCAD 15 programming environment.

Globally, significant attention is being paid to the development and safety of high-speed electric rolling stock (specifically, electric trains and high-speed electric trains), and the improvement of their spring suspension systems using modern means and equipment, utilizing advanced technologies. In developed countries such as the USA, England, France, Spain, Germany, Japan, China, Russia, and elsewhere, special attention is being paid to improving controlled spring suspension systems when designing and building new electric trains and high-speed electric trains [1, 2, 3, 4].

Ultimately, the history of the development of the mechanical part of high-speed electric trains has led to the fact that the currently existing high-speed rolling stock, as a rule, has bogies with two stages of spring suspension, each of which contains elastic and dissipative elements. In the current state of the railway track, to ensure good smooth running of high-speed electric trains, passenger cars and locomotives, it is necessary to have a sufficiently "soft" spring suspension. For this purpose, pneumatic spring suspension is often used. Designs with such suspension make it easier to ensure the standard value of static deflection of 200 mm for passenger cars and rolling stock designed for speeds of 250 km/h and more. In addition, such suspension has both elastic and dissipative properties, i.e. no special damper is required [4].

Research has been conducted and is being conducted on this topic by leading scientists worldwide such as S.A. Brebbia (Wessex Institute of Technology, UK), G.M.



Carlomagno (University of Naples di Napoli, Italy), A. Varvani-Farahani (Ryerson University, Canada), S.K. Chakrabarti (USA), S. Hernandez (University of La Coruna, Spain), S.-H. Nishida (Saga University, Japan). Authoritative scientific schools and prominent scientists in the CIS countries from MIIT, PGUPS, MAI, VNIIZhT, JSC VNIKTI, JSC Russian Railways, etc. have worked on these issues. A significant contribution to solving many complex problems and checking theoretical conclusions related to the study of the oscillation processes of the spring suspension of the rolling stock was made by the Russian Research Institute of Railway Transport (CNII MPS) and the Russian Research Institute of Railcar Building (NIIV); there, along with theoretical studies, a large number of experimental studies (bench and full-scale ones) were conducted. In Uzbekistan, the academician of the Academy of Sciences of the Republic of Uzbekistan, professor, doctor of Technical Sciences Glushchenko A.D., professors Fayzibaev Sh.S., Khromova G.A., Shermukhamedov A.A., D.O. Radjibayev and their students dealt with the problems of optimizing the systems of spring suspension of rolling stock [5-10].

A mathematical model for studying the horizontal vibrations of the AFROSIAB high-speed electric train's multiple unit car during its curved travel was presented in [5]. The calculation scheme is shown in Figure 1.

Calculation of the track loading due to the wheel sets of the AFROSIAB high-speed electric train's multiple unit car (lead multiple unit car) during its curved travel (in a curved section of track).

Taking into account the assumptions made in [5,6,7,8,9,10], we use the following equations for elastic vibrations of the model sections (Figure 1):

- the first $y_1(\ell, t)$, characterizing the vibrations of the masses of the wheelsets relative to the surface of the outer rail

$$\mu_1 \frac{\partial^2 y_1}{\partial t^2} + i_1 \frac{\partial^4 y_1}{\partial l^2 \partial t^2} + EI_1 \frac{\partial^4 y_1}{\partial l^4} + K_1 y_1 = \left(q_{\pi} - \frac{F_0}{l_0} \right) \cos \left(\frac{2\pi l}{l_0} + \frac{2\pi v_k t}{l_0} \right), \quad (1)$$

- the second $y_2(\ell, t)$, characterizing the vibrations of the sections of the motor-car bogies of the electric train car relative to the outer rail

$$\mu_2 \frac{\partial^2 y_2}{\partial t^2} + i_2 \frac{\partial^4 y_2}{\partial l^2 \partial t^2} + EI_2 \frac{\partial^4 y_2}{\partial l^4} + K_2 y_2 = n_1(t, \ell), \quad (2)$$

- the third $y_3(\ell, t)$, characterizing the oscillations of the sections of the electric train car body relative to the outer rail

$$\mu_3 \frac{\partial^2 y_3}{\partial t^2} + i_3 \frac{\partial^4 y_3}{\partial l^2 \partial t^2} + EI_3 \frac{\partial^4 y_3}{\partial l^4} + K_3 y_3 = n_2(t, \ell), \quad (3)$$

where t is the process time;

ℓ is the coordinate of the section placement along an arc of radius R , measured from point 1 (the "conditional" hinge in the front automatic coupling of the section of the lead multiple unit car), the maximum value of $\ell = \ell_K$;

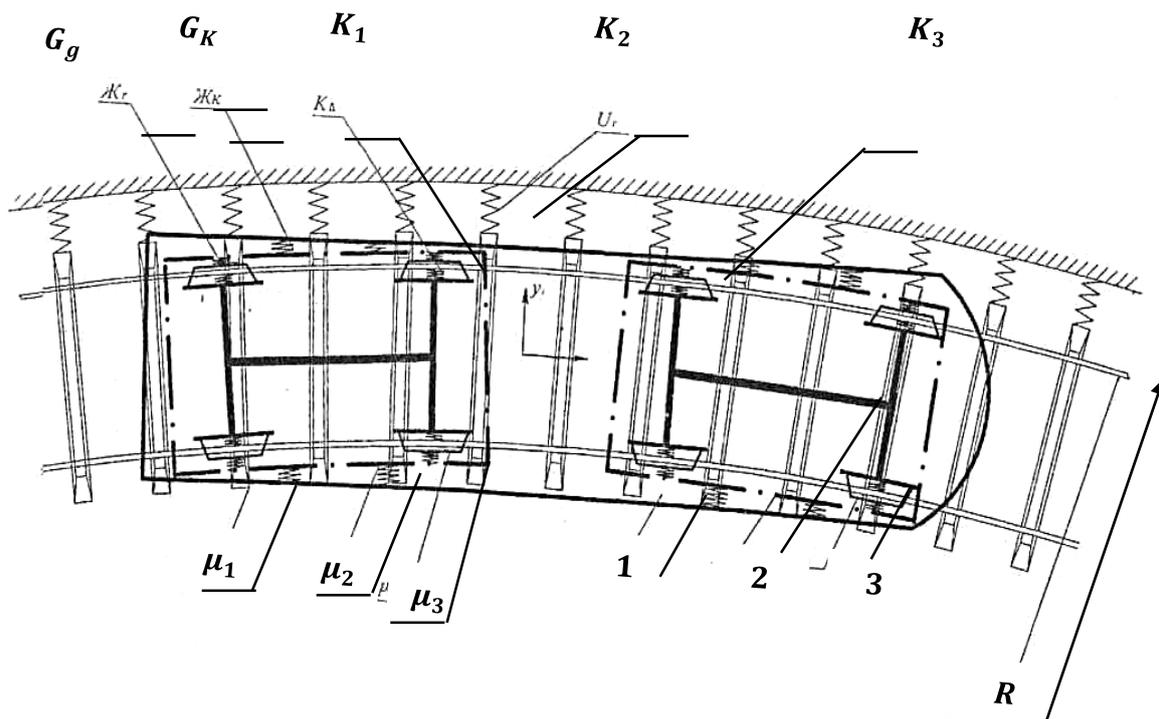


Figure 1. Calculation scheme for studying the oscillation process of the spring suspension of the AFROSIAB high-speed electric train in the horizontal plane as it moves along curves in the track.

$F_T = N_0 \cdot f_T$ - the sliding friction force of the wheel of the lead motor car, loaded with a static vertical load N_0 along the surface of the outer rail;

f_T - coefficient of sliding friction of the wheel on the rail in the direction $y_1(\ell, t)$;

$n_1(t, \ell)$ and $n_2(t, \ell)$ - intensities of inertial forces of the first and second models;

μ_1 and μ_2 - intensity of the masses of the bogies and body;

K_1 and K_2 - the intensity of elastic connections between the outer rail and the bogie frames, between the bogies and the body;

G_g and G_K - horizontal and torsional stiffness.

We will take the initial data for the numerical calculation as follows:

2000 sleepers are laid in the curve with a pitch $\ell_{sh} = 0,5$ m per 1 km of track, $\ell_T = 10.65$ m is the average distance between the axes of rotation of the wheel sets for the lead multiple unit car, the length of the lead car $\ell_{\text{л}} = 20.748$ m, the weight of the multiple unit car (lead multiple unit car) of the AFROSIAB high-speed electric train $G_L = 66.4$ t, the weight of one bogie with wheelsets of type B-6234 $G_T = 13.94$ t, the weight of one wheelset $G_{KP} = 1.3$ t, the calculated speed of the lead multiple unit car in the curve $V_p = 92 = 92$ km/h, the radius of the curve $R = 500$ m.

1. Weight calculation.

Car body weight $G_K = G_L - 2 \cdot G_T = 38,52$ t.

Weight of the bogie without wheelsets

$G_T^0 = G_T - 2 \cdot G_{KP} = 13,94 - 2 \cdot 1,3 = 11,34$ t.

2. Determine the mass intensity.



- the mass intensity of the body $\mu_K = \frac{G_K}{\ell_L} = \frac{38520}{20.748} = 1856.565 \text{ kg/m}$;
- intensity of the mass of bogies $\mu_T = \frac{G_T}{\ell_T} = \frac{13940}{10.65} = 1308.92 \text{ kg/m}$;
- intensity of the mass of wheelsets
- $\mu_{KP} = \frac{4 \cdot G_{KP}}{\ell_L} = \frac{4 \cdot 1300}{20.748} = 250.627 \text{ kg/m}$;
- rail mass intensity (P65) $\mu_P = \frac{G_P}{g} = \frac{65}{9.81} = 6.626 \text{ kg/m}$.

3. We calculate the frequencies of natural oscillations using the MATHCAD 15 programming environment:

- the first model

$$P_{11} = \sqrt{\frac{2 \cdot 1,184 \cdot 10^5 \cdot 171.227}{35.4^2} \left(1 + \sqrt{1 - \frac{3.945 \cdot 10^4 \cdot 35.4^2}{1,184 \cdot 10^5 \cdot 171.227^2}}\right)} = 253.9c^{-1}$$

$$P_{12} = \sqrt{\frac{2 \cdot 1,184 \cdot 10^5 \cdot 171.227}{35.4^2} \left(1 - \sqrt{1 - \frac{3.945 \cdot 10^4 \cdot 35.4^2}{1,184 \cdot 10^5 \cdot 171.227^2}}\right)} = 15.2c^{-1}$$

- the second model

$$P_{21} = \sqrt{\frac{2 \cdot 1,184 \cdot 10^5 \cdot 462.596}{168.8^2} \left(1 + \sqrt{1 - \frac{3.025 \cdot 10^4 \cdot 168.8^2}{1,184 \cdot 10^5 \cdot 462.596^2}}\right)} = 87.3c^{-1}$$

$$P_{22} = \sqrt{\frac{2 \cdot 1,184 \cdot 10^5 \cdot 462.596}{168.8^2} \left(1 - \sqrt{1 - \frac{3.025 \cdot 10^4 \cdot 168.8^2}{1,184 \cdot 10^5 \cdot 462.596^2}}\right)} = 8.1c^{-1}$$

- the third model

$$P_{31} = \sqrt{\frac{2 \cdot 1,184 \cdot 10^5 \cdot 742.51}{271^2} \left(1 + \sqrt{1 - \frac{1.261 \cdot 10^4 \cdot 271^2}{1,184 \cdot 10^5 \cdot 742.51^2}}\right)} = 69.1c^{-1}$$

$$P_{32} = \sqrt{\frac{2 \cdot 1,184 \cdot 10^5 \cdot 742.51}{271^2} \left(1 - \sqrt{1 - \frac{1.261 \cdot 10^4 \cdot 271^2}{1,184 \cdot 10^5 \cdot 742.51^2}}\right)} = 4.1c^{-1}$$

Based on experimental data, the values of the highest and lowest natural frequencies lie within the following ranges:

- for the first model, within $p_{11} = (185 \div 255) c^{-1}$, $p_{12} = (8.4 \div 17.8) c^{-1}$;
- for the second model, within в пределах $p_{21} = (85 \div 95) c^{-1}$, $p_{22} = (6.4 \div 14.2) c^{-1}$;
- for the third model, within в пределах $p_{31} = (60 \div 70) c^{-1}$, $p_{32} = (4.0 \div 6.8) c^{-1}$;

The calculated values of the natural frequencies for each model are within the experimental data range for high-speed electric trains, confirming the accuracy of the calculations.

As can be seen from the obtained results, the equivalent vertical and transverse horizontal stiffness of the upgraded spring suspension remained unchanged compared to the corresponding stiffnesses of the existing spring suspension. This means that the vertical and transverse horizontal dynamics of high-speed electric trains will not change as a result of using the upgraded spring suspension. However, a positive effect is that the spring stiffness of the upgraded spring suspension in the horizontal transverse direction

is doubled, which means the offset (drift) of the upper spring coil relative to the lower coil will decrease.

The system of differential equations (1)÷ (3) is solved by the Gauss matrix method using the MathCAD 15 programming environment. As a result, the amplitude-frequency response of the “car body-bogie-track” system is investigated considering the influence of spring suspension using for example the high-speed electric train AFROSIAB, moreover, the first and second bogies vibrate with different amplitudes, and the wheel pairs also vibrate differently. Graphs are plotted for the bouncing z_k and galloping $\varphi_{yk} \approx 0$ oscillations of the car body of the electric train, and for the bouncing oscillations of wheelsets z_T (Figure 2).

As a result, we have developed an analytical and numerical model using the Gauss method, which allows us to analyze the amplitude-frequency spectrum of vertical oscillations of the AFROSIAB electric train car model, and to determine the influence of hydraulic and pneumatic damping in the system.

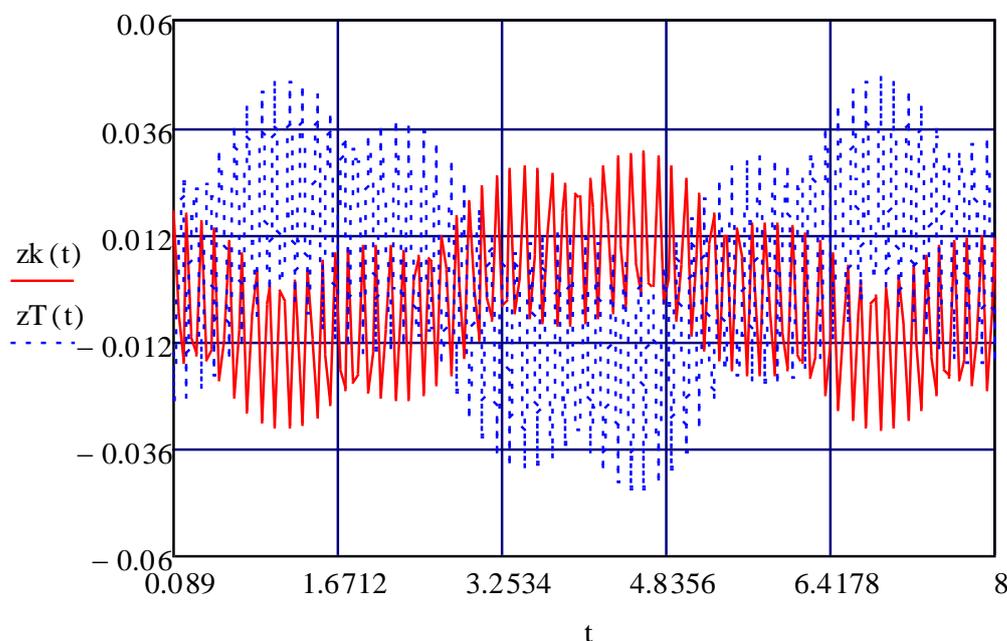


Figure 2. Graph of bouncing $z_k(t)$ oscillations of the body of the electric train car and recoiling oscillations of wheelsets $z_T(t)$.

Based on the presented mathematical model using formulas (1)÷(11), considering the actual dimensions of the spring suspension of the high-speed electric train AFROSIAB, a numerical calculation was performed to justify rational parameters and to build the amplitude-frequency response of the “track-wheel-bogie-body” system.

Conclusion

Based on the theoretical and numerical studies conducted, the following general conclusions can be drawn:

1. We developed an algorithm and a program for the MathCad 15 programming environment to describe the vertical oscillations of the high-speed electric train AFROSIAB. We then conducted numerical studies for the proposed mathematical model.



2. As a result, we developed an analytical and numerical model using a method similar to the Gauss method. This model allows us to analyze the amplitude-frequency spectrum of vertical oscillations of the high-speed electric train AFROSIAB.

3. Based on the numerical results, we identified the most dangerous zones, where the amplitudes of vertical oscillations are most considerable. It is evident (see Figure 3) that different vertical oscillations of the wheelsets z_{kp1} and z_{kp2} cause significant bouncing oscillations of the first and second bogies z_{T1} , z_{T2} . These oscillations are then transferred to the car body of the high-speed electric train AFROSIAB within the "track-wheel-bogie-body" system, leading to a deterioration in its smoothness. Therefore, it is necessary to implement pneumatic spring suspension in the central stage to optimize the functions of the spring suspension and enhance its elastic-dissipative properties at high speeds of the electric rolling stock.

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