SOME NON-STANDART PROBLEMS OF ANALYTIC GEOMETRY

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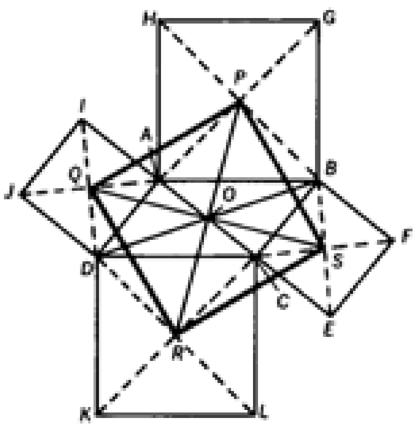
Annotation
In this article, the solutions of some problems that belong to the analytical geometry part of geometry, are not included in the school geometry course and the higher geometry course, and are considered non-standard. That is, the solution to the problematic questions related to rectangles with a relatively high level of complexity was found analytically. The considered issues serve to develop the geometric worldview of students who want to master analytical geometry in depth.
Keywords. non-standart problems, analytic geometry, rectangles, bissektor, parallelogram, The quadrilateral, fashion similarly, rhombus.

Introduction. In this work, the problems related to the internal properties of a square, a square and a parallelogram were solved. The goal of finding a solution to the problems presented in the article in an analytical way is set, and for this, the task of using the necessary fundamental properties is assigned. The solved problems are completely non-standard and are relevant for the development of geometric imagination for students of mathematics in higher education.



Difenation. A triangle is a simple closed curve or polygon which is created by three line-segments. In geometry, any three points, specifically non-collinear,form a unique triangle and separately, a unique plane [1:179].

The SAS Similarity Theorem. Given a correspondence be-tween two triangles. If two pairs of corresponding sides are proportional, and the included angles are congruent, then the correspondence is a similarity[1:189].

The ASA Similarity Theorem. If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent[5:220].

Theorem-1.

If a square is drawn externally on each side of a parallelogram, prove that:

(a) The quadrilateral, determined by the centers of these squares, is itself a square;

(b) The diagonals of the newly formed square are concurrent with the diagonals of the original parallelogram.

Proof: *(a)* $ABCD$ is parallelogram. Points P, Q, R and S are the centres of the four squares, $DAIJ$, $DCLK$, and $CBFE$, respectively (fig. S-1). $PA = DR$ and $AQ = QD$ (each is one-half a diagonal). Also, $\angle ADC$ is supplementary to $\angle DAB$, and $\angle IAH$ is supplementary to $\angle DAB$ (since $\angle AID \cong \angle HAB \cong$ right angle). Therefore, $\angle ADC \cong \angle IAH$.

Since, $m\angle RDC = m\angle QDA = m\angle HAP = m\angle QAI = 45^0$, $\angle RDQ \cong \angle QAP$. Thus, $\triangle RDQ \cong \triangle PAQ$ (s.a.s), and $QR = QP$. In a fashion similarly, it may be proved that $QP = PS$ and $PS = RS$. Therefore, $PQRS$ is a rhombus.

Since, $\triangle RDQ \cong \triangle PAQ$, $\angle DQR \cong \angle AQP$; therefore, $\angle PQR \cong \angle DQA$ (by addition). Since, $\angle DQA$ - right angle, (S-1)

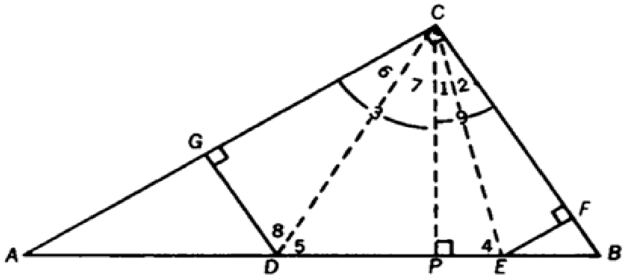
and $PQRS$ a is square.

(b) To prove that the diagonals of $PQRS$ are concurrent with the diagonals of Parallelograms $ABCD$, we must prove that a diagonal of the square and a diagonal of the parallelogram bisect each other. In other words, we prove that the diagonals of the square and the diagonals of the parallelogram all share the same midpoint, (i.e., point O). $\angle BAC \cong \angle ACD$, and $m\angle PAB = m\angle RCD = 45^0$; therefor, $\angle PAC \cong \angle RCA$. Since, $\angle AOP \cong \angle COR$ and $AP = CR$, $\triangle AOP \cong \triangle COR$ (s.a.a).

Thus, $AO = CO$, and $PO = RO$. Since, $\overrightarrow{DB}$ passes through the midpoint of $\overrightarrow{AC}$, and similarly $\overrightarrow{QS}$ passes through the midpoint of $\overrightarrow{PR}$, and since $\overrightarrow{AC}$ and $\overrightarrow{PR}$ share the same midpoint (i.e., O), we have shown that $\overrightarrow{AC}, \overrightarrow{PR}, \overrightarrow{DB},$ and $\overrightarrow{QS}$ are concurrent (i.e., all pass through point O).

Theorem-3.

In right $\triangle ABC$, *with right angle at* C, $BD = BC$, $AE = AC$, $\overline{EF} \perp \overline{BC}$, *and* $\overline{DG} \perp \overline{AC}$. *Prove that* $DE = EF + DG$.



(S-2)

Proof. Draw $\overline{CP} \perp \overline{AB}$, also draw $\overline{CE}$ and $\overline{CD}$ (Fig. S-4).

$$m\angle 3 + m\angle 1 + m\angle 2 = 90^\circ$$

$$m\angle 3 + m\angle 1 = m\angle 4 \ (\#5)$$

By substitution,

$$m\angle 4 + m\angle 2 = 90^\circ$$

but in right $\triangle CPE$, $m\angle 4 + m\angle 1 = 90^\circ$.

Thus, $\angle 1 \cong \angle 2$ (both are complementary to $\angle 4$), and right $\triangle CPE \cong right\ \triangle CFE$, and $PE = EF$.

Similarly,

$$m\angle 9 + m\angle 7 + m\angle 6 = 90^\circ$$

$m\angle 9 + m\angle 7 = m\angle 5$.

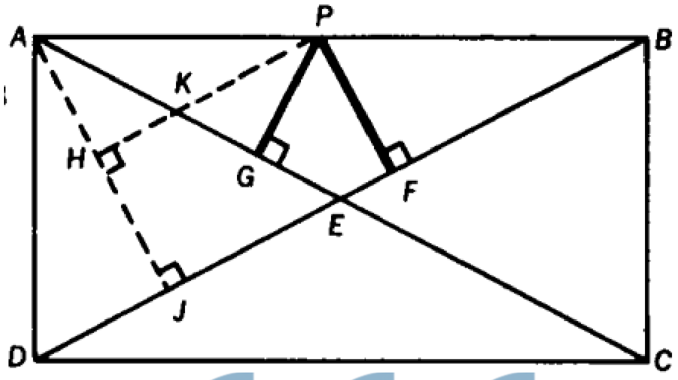
By substitution, $m\angle 5 + m\angle 6 = 90^\circ$. However, in right $\triangle CPD$,

$$m\angle 5 + m\angle 7 = 90^\circ$$

Thus, $\angle 6 \cong \angle 7$ (both are complementary to $\angle 5$), and right

$\triangle CPD \cong right \triangle CGD$ and $DP = DG$.
Since $DE = DP + PE$, we get $DE = DG + EF$.

Theorem-4.
Prove that the sum of the measures of the perpendiculars from any point on a side of a rectangle to the diagonals is constant.



(S-3)

Proof. Let P be any point on side $\overline{AB}$ of rectangle ABCD (Fig. S-5).
$\overline{PG}$ and $\overline{PF}$ are perpendiculars to the diagonals. Draw $\overline{AJ}$ perpendicular to $\overline{DB}$, and then $\overline{PH}$ perpendicular to $\overline{AJ}$. Since $PHJF$ is a rectangle (a quadrilateral with three right angles),we get $PF = HJ$. Since $\overline{PH}$ and $\overline{BD}$ are both perpendicular to $\overline{AJ}$, $\overline{PH}$ is parallel to $\overline{BD}$. Thus, $\angle APH \cong \angle ABD$. Since, $AE = EB$, $\angle CAB \cong \angle ABD$. Thus, by transitivity, $\angle EAP \cong \angle APH$; also in $\triangle APK$, $AK = PK$. Since $\angle AKH \cong \angle PKG$, right $\triangle AHK \cong right \triangle PGK$ (S.A.A.). Hence, $AH = PG$ and, by addition, $PF + PG = HJ + AH = AJ$, a constant.

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