PLANIMETRIYANING MASHXUR TEOREMASI. STYUART TEOREMASI

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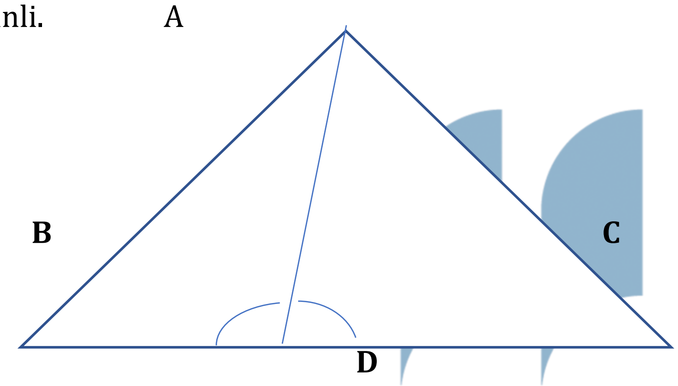
Annotatsiya: Ushbu maqolada biz Evklid geometriyasining asosiy teoremalaridan biri-Styuart teoremasini ko'rib chiqamiz, u buni isbotlagan ingliz matematigi M.Styuart sharafiga shunday nom olgan. Shuningdek, biz teoremaning masalalar yechishga tadbiqini keltirib o'tamiz. Shuningdek bu teoremadan biz turli formulalarni isbotlashda foydalanishimiz ham keltirilib o'tilgan.

Kalit so'zlar: teorema, isbot, ta'rif.

Bizga ABC uchburchak berilgan bo'lsin va bu uchburchak uchun quyidagi teorema o'rinli. A



Teorema. Agar D nuqta ABC uchburchakning BC tomonida yotsa, u holda:

$AD^2 * a = BD * b^2 + DC * c^2 - BD * DC * a$.

Isbot. Mos holda A burchak qarshisidagi tomonni a, B burchak qarshisidagi tomonni b va C burchak qarshisidagi tomonni c bilan belgilaymiz. Agar $\angle BDA = \alpha$ bo'lsa, $\angle ADC = 180° - \alpha$ bo'ladi. Shunga ko'ra $\triangle BDA$ va $\triangle ADC$ lar uchun kosinuslar teoremasini qo'llyamiz.

$$c^2 = BD^2 + AD^2 - 2 * BD * AD * cos\alpha$$

$$b^2 = DC^2 + AD^2 - 2 * AD * DC * \cos(180° - \alpha)$$

endi birinchi ifodani DC ga ikkinchi ifodani esa BD ga ko'paytiramiz.

$$DC * c^2 = DC * BD^2 + DC * AD^2 - 2 * DC * BD * AD * cos\alpha$$

$$BD * b^2 = BD * DC^2 + BD * AD^2 - 2 * BD * AD * DC * \cos(180° - \alpha)$$

$\cos(180° - \alpha) = -cos\alpha$ ekanligidan, birinchi ifodaga ikkinchi ifodani qo'shsak quyidagi hosil bo'ladi.

$$DC * c^2 + BD * b^2 = DC * BD^2 + DC * AD^2 + BD * DC^2 + BD * AD^2 \leftrightarrow$$

$$(DC + BD) * AD^2 = DC * c^2 + BD * b^2 - DC * BD * (BD + DC)$$

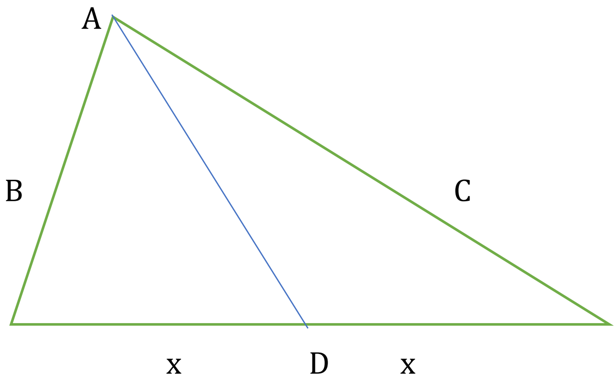
BD+DC=BC=a

Demak, $AD^2 * a = BD * b^2 + DC * c^2 - BD * DC * a$ formula o'rinli ekanligi kelib chiqadi.

Teoremaning qo'llanilishi

Styuart teoremasidan uchburchakning medianalari va bissektrisalarini topish uchun formulalar olinishi mumkin va shuningdek, uchburchakka tashqi chizilgan aylana markazi bilan uchburchak og'irlik markazi orasidagi masofani ham topishimiz mumkin.

1. Mediana uzunligini keltirib chiqarish.



AB=c, BC=a, AC=b

Yuqoridagi teoremadan foydalansak, quyidagilar hosil bo'ladi.

$AD^2 * a = BD * b^2 + DC * c^2 - BD * DC * a$.

BD=DC ekanligidan, $AD^2 * a = BD * (b^2 + c^2) - BD^2 * a$. $BD = \frac{a}{2}$

$$AD^2 * a = \frac{a}{2} * (b^2 + c^2) - \left(\frac{a}{2}\right)^2 * a$$

$2 * AD^2 = (b^2 + c^2) - a^2 \quad AD = m_a$

$$m_a = \frac{1}{2} * (\sqrt{2 * (b^2 + c^2) - a^2})$$

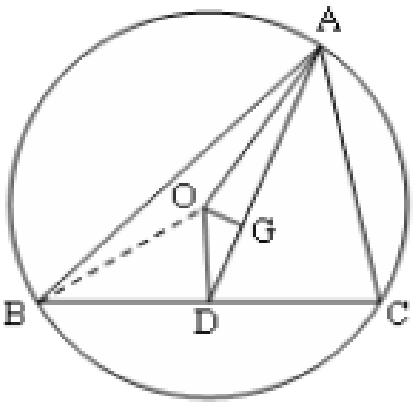
Demak, har bir tomonga tushurilgan mediana uzunligi mos ravishda,

a tomonga tushurilsa, $m_a = \frac{1}{2} * \sqrt{2 * (b^2 + c^2) - a^2}$,

b tomonga tushurilsa, $m_b = \frac{1}{2} * \sqrt{2 * (a^2 + c^2) - b^2}$,

c tomonga tushurilsa , $m_c = \frac{1}{2} * \sqrt{2 * (b^2 + a^2) - c^2}$ formulalar orqali topiladi.

1-masala: Uchburchakka tashqi chizilgan aylana markazi bilan uchburchak og'irlik markazi orasidagi masofa topilsin.



Yechim. ABC uchburchak BC tomonining o'rta nuqtasi D va izlangan masofa OG bo'lsin. OBD uchburchakdan: $OD^2 = R^2 - \frac{a^2}{4}$ $va\ AD = m_a$, Styuart teoremasini tadbiq etib, $m_a = \frac{1}{2} * \sqrt{2 * (b^2 + c^2) - a^2}$ formula bo'yicha hisoblansa:

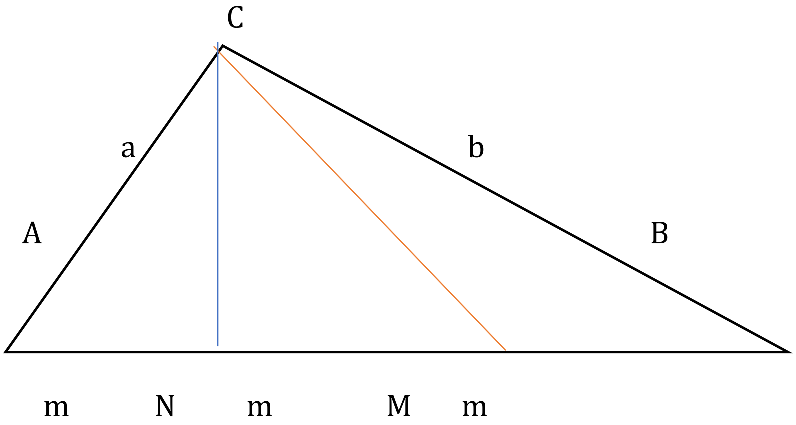
$OG^2 * AD = OA^2 * DG + OD^2 * AG - AD * AG * DG$ yoki

$OG^2 * m_a = R^2 * \frac{1}{3} m_a + (R^2 - \frac{a^2}{4}) * \frac{2}{3} m_a - m_a * \frac{1}{3} m_a * \frac{2}{3} m_a$, bundan,

$$OG^2 = \frac{1}{3}R^2 + \frac{2}{3}\left(R^2 - \frac{a^2}{4}\right) - \frac{2}{9} * m_a^2 = R^2 - \frac{a^2+b^2-c^2}{9},$$

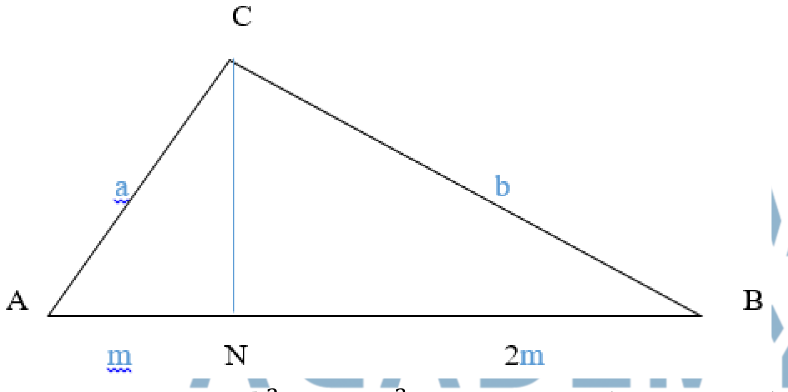
demak: $OG = \frac{1}{3}\sqrt{9R^2 - (a^2 + b^2 - c^2)}$.

Misol.



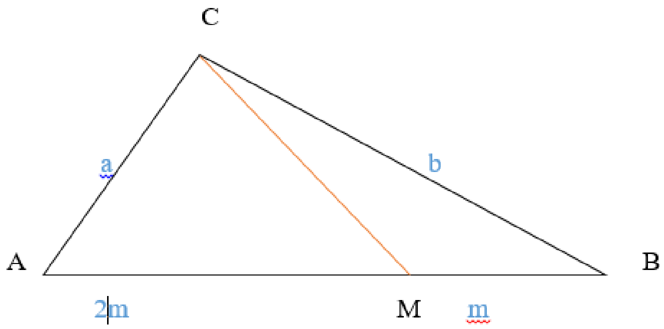
Yuqoridagi berilgan uchburchakda quyidagicha munosabatlar mavjud: berilgan uchburchak to‘g‘ri burchakli uchburchak bo‘lib $\angle ACB = 90^{\circ}$, CM=4, CN=3 va AN=NM=MB=m=?

ABC to‘g‘ri burchakli uchburchak ekanligidan Pifagor teoremasiga ko‘ra $a^2 + b^2 = 9m^2$ ga teng. Quyidagi uchburchak uchun Styuart teoremasini qo‘llaymiz:



$$b^2 * m + a^2 * 2m = 3m * (m * 2m + 9)$$

2-tomondan,



$$b^2 * 2m + a^2 * m = 3m * (m * 2m + 16)$$

Ikkala ifodani soddalashtiramiz,

$$m(b^2 + 2a^2) = 3m * (2m^2 + 9)$$

$$m(a^2 + 2b^2) = 3m * (2m^2 + 16)$$

Ikkala tarafdan ham m ni qisqartirib, chap taraflarni o‘zaro qo‘shamiz.

$3(b^2 + a^2) = 3m * (2m^2 + 9 + 2m^2 + 16)$

$a^2 + b^2 = 9m^2$ *ekanligidan foydalansak*, $9m^2 = 4m^2 + 25$

$5m^2 = 25 \quad m = \sqrt{5}$

Javob: $m = \sqrt{5}$

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