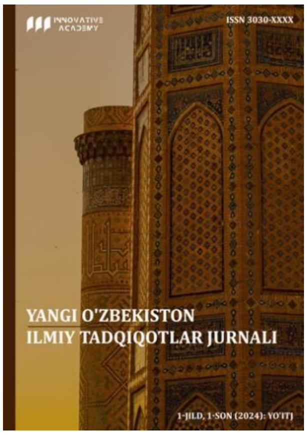

CHO'KMA HOSIL BO'LADIGAN SUSPENZIYALARNI FILTRLASH TENGLAMALARI

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ABSTRACT

Ishda keyk-qatlam hosil bo'ladigan holatda suvpenziyalarni sızıshining o'qqa nisbatan simmetrik masalasi sonli yechilgan. Modelda turli parametrlarning keyk-qatlam bo'ylab kompressiyaviy va suyuqlik bosimiga hamda keyk-qatlam qalinligining o'sishiga ta'siri o'rganilgan

Silindr koordinatalaridagi suyuq va qattiq fazalar uchun uzluksizlik tenglamalari quyidagicha [4-7]:

$$\frac{\partial \varepsilon}{\partial t} + \frac{1}{2\pi r}\frac{\partial q_\ell}{\partial r} = 0, \qquad (1)$$

$$\frac{\partial \varepsilon_s}{\partial t} + \frac{1}{2\pi r}\frac{\partial q_s}{\partial r} = 0, \qquad (2)$$

bu yerda t - vaqt, r - radial koordinata, q_ℓ - suyuq fazaning tezligi, q_s - qattiq fazaning tezligi, ε - g'ovaklik, ε_s - qattiq zarrachalar ulushi.

(1) va (2) tenglamalardan quyidagiga kelamiz

$$\frac{\partial q_\ell}{\partial r} + \frac{\partial q_s}{\partial r} = 0.$$

Bu tenlikdan

$$q_\ell + q_s = q_{out}, \qquad (3)$$

kelamiz. Bu erda q_{out} - umumiy filtrlash tezligi, koordinata r dan bog'liq emas..

Silindrik koordinatalarda Darsi qonuni [5,6] quyidagi ko'rinishga ega

$$q_\ell - \frac{\varepsilon}{\varepsilon_s} q_s = -2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}, \qquad (4)$$

p_ℓ - suyuq faza bosimi.

(4) ni r bo'yicha deffirensialllab

$$\frac{\partial q_\ell}{\partial r} = -\frac{\partial}{\partial r}\quad 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r} - \frac{\varepsilon}{\varepsilon_s} q_s \qquad (5)$$

kelamiz. Cho'kma va filtr elementi chegarasida suzpenziya qattiq fazasining tezligi nolga teng ekanligidan $\left(q_s\big|_{r=R} = 0\right)$, cho'kmaning ixtiyoriy nuqtalarida quyidagi munosabtaga kelamiz

$$q_\ell + q_s = q_{out} = -\left. 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\right|_{r=R} = \frac{-p_{\ell m}}{\mu R_m}, \quad (6)$$

bu erda R filtr elementining tashqi radiusi, $p_{\ell m}$ filtr elementidagi bosim va R_m - $r=R$ dagi nisbiy qarshiligi.

Tenglama (3) va (4) dan

$$q_s = \varepsilon_s \left(2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r} - \left. 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\right|_{r=R} \right) \quad (7)$$

munosabatga kelamiz.

(7) ni (5) ga qo‘ysak quyidagi tenglikka kelamiz

$$\frac{\partial q_\ell}{\partial r} = -\frac{\partial}{\partial r}\left(\varepsilon_s 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r} + (1-\varepsilon_s) \left. 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\right|_{r=R} \right).$$

Yoki

$$\frac{\partial q_\ell}{\partial r} = -\frac{\partial}{\partial r}\left(\varepsilon_s 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r} \right) - \left. 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\right|_{r=R} \frac{\partial \varepsilon_s}{\partial r}.$$

(1), (2) dan foydalanib, oxirgi tenglikni quyidagi shakla yoza olamiz

$$\frac{\partial \varepsilon_s}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left(\varepsilon_s r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r} \right) - \frac{q_{вых}}{2\pi}\frac{1}{r}\frac{\partial \varepsilon_s}{\partial r}, \quad (8)$$

bu yerda $q_{out} = \left. -2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\right|_{r=R}$ filtrda chiqishdagi filtrat tezligi.

(8) tenglama *silindr koordinatalarida cho‘kma qatlamini hosil bo‘ladigan suspenziyalarni filtrlash*ning asosiy tenglamasidir. Bu tenglama $r = R_L(t)$ nuqtadagi qo‘zg‘aluvchan chegarani aniqlash tenglamasi hamda boshlang’ich va chegara shatrlar bilan birgalikda yechiladi.

Cho‘klma qatlamining xususiyatlari bir necha parametrlar bilan ifodalanadi [1]: ε_s - qattiq zarrachalar ulushi, k - filtrning o’tkazuvchanligi. [1,2] adabiyotlardan foydalanib, ushbu parametrlarning p_s suyuqlik bosimining funksiyasi ekanligini ko‘rish mumkin:

$$\varepsilon_s = \varepsilon_s^0 \left(1 + \frac{p_s}{p_A}\right)^{\beta}, \quad (9)$$

$$k = k^0 \left(1 + \frac{p_s}{p_A}\right)^{-\delta}, \quad (10)$$

bu erda ε_s^0, k^0, α^0 - mos ravishda $p_s = 0$ nuqtadagi ε_s, k α qiymatlari, p_A - xarakteristik bosim, β , , δ $n = \delta - \beta$ – doimiy qiymatlar.

(9) va (10) lardan foydalanib (8) ni p_s dan bog‘lik tenglamaga kelamiz

$$\frac{\beta\varepsilon_s^0}{p_A}\left(1+\frac{p_s}{p_A}\right)^{\beta-1}\frac{\partial p_s}{\partial t}=-\frac{\varepsilon_s^0 k^0}{\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta} r\frac{\partial p_\ell}{\partial r}\right)-$$
$$-\frac{\beta\varepsilon_s^0}{p_A}\left(1+\frac{p_s}{p_A}\right)^{\beta-1}\frac{q_{out}}{2\pi}\frac{1}{r}\frac{\partial p_s}{\partial r}, \quad (11)$$

bu yerda

$$q_{\ell m}=-2\pi r\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)\frac{\partial p_\ell}{\partial r}\bigg|_{r=R}=2\pi r\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)\frac{\partial p_s}{\partial r}\bigg|_{r=R}.$$

[2] adabiyotgan foydalanib, p_ℓ va p_s o'rtasidagi bog'lanish munosabatini olamiz:

$$dp_\ell+dp_s=0, \quad (12)$$

yoki

$$\frac{\partial p_\ell}{\partial p_s}=f'=-1. \quad (13)$$

$f'=-1$ ekanligidan (11) quyidagi koʻrinishga keladi

$$\frac{\beta\varepsilon_s^0}{p_A}\left(1+\frac{p_s}{p_A}\right)^{\beta-1}\frac{\partial p_s}{\partial t}=\frac{\varepsilon_s^0 k^0}{\mu}\frac{1}{r}\frac{\partial}{\partial r}\left(\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta} r\frac{\partial p_s}{\partial r}\right)-$$
$$-\frac{\beta\varepsilon_s^0}{p_A}\left(1+\frac{p_s}{p_A}\right)^{\beta-1}\frac{q_{out}}{2\pi}\frac{1}{r}\frac{\partial p_s}{\partial r} \quad (14)$$

koʻrinishga keladi

Endi cho'kma qatlamning o'sishini tavsiflovchi qoʻzgʻaluvchan chegara tenglamasini (Stefan shartini) keltirib chiqaraylik. R_L - cho'kma qatlamning qalinligi boʻlib, qoʻzgʻaluvchan chegara ifodalaydi va jarayon boshlangandan o'suvchi boʻlib, vaqtning funksiyasi sifatida cho'kma qatlamning qalinligi bildiradi. Choʻkma yuzasidagi massaning saqlanish qonunidan foydalanib bogʻlanish tenglamasini toppish mumkin. Quydagi belgilashlarni kiritamiz: $q_\ell\big|_{L^+}$ va $q_\ell\big|_{L^-}$ mos ravishda $x=R_L^+$ (choʻkma-suspeziya chegarasining suspenziya sohasidagi) va $x=R_L^-$ (choʻkma-suspeziya chegarasining choʻkma sohasidagi) dagi suyuqlik tezliklari. $\varepsilon_s\big|_{L^+}$ va $\varepsilon_s\big|_{L^-}$ mos ravishda $x=R_L^+$ (choʻkma-suspeziya chegarasining suspenziya sohasidagi) va $x=R_L^-$ (choʻkma-suspeziya chegarasining choʻkma sohasidagi) dagi choʻkmadagi qattiq fazalar ulushi. Agar δt vaqt monetida choʻkma qalinligi δR_L ga oshsa quyidagi suyuqlik uchun massa saqlanish tenglamasini olamiz

$$\left[\left(q_\ell\big|_{L^-}-q_\ell\big|_{L^+}\right)\right]\delta t=\left[\left(1-\varepsilon_s\big|_{L^-}\right)-\left(1-\varepsilon_s\big|_{L^+}\right)\right]2\pi R_L\delta R_L.$$

Oxirgi munosabatdan choʻkma qatlamining oʻsishi tenglamasini olamiz:

$$2\pi R_L\frac{dR_L}{dt}=\frac{q_\ell\big|_{L^-}-q_\ell\big|_{L^+}}{\varepsilon_s\big|_{L^+}-\varepsilon_s\big|_{L^-}}, \quad (15)$$

bu erda $r=R_{L^-}$ nuqtada zarralarning siqilishi kuzatilmaydi, ya'ni nolga teng ekanligidan

$\varepsilon_s\big|_{R_{L^-}}$ ni ε_s^0 teng deb olish mumkin. $\varepsilon_s\big|_{R_{L^+}}$ esa suspenziyada qattiq zarralarning kosentrasiyasi ε_{s_0} ga teng bo‘ladi. U holda (15) ni qayta ypzib olish mumkin.

$$2\pi R_L \frac{dR_L}{dt} = \frac{q_\ell\big|_{L^-} - q_\ell\big|_{L^+}}{\varepsilon_{s_0} - \varepsilon_s^0}. \tag{16}$$

Darsi qonunidan quydagi munosabatlarga olamiz

$$q_\ell\big|_{L^+} + q_s\big|_{L^+} = q_\ell\big|_{L^-} + q_s\big|_{L^-} = q_{\ell m} = -2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{r=R}. \tag{18}$$

$x = R_L^-$ nuqatada

$$q_\ell\big|_{L^-} - \frac{1-\varepsilon_s^0}{\varepsilon_s^0} q_s\big|_{L^-} = -2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{x=R_L^-}. \tag{19}$$

(3) e’tiborga olsak

$$q_s\big|_{L^-} = -q_\ell\big|_{L^-} - 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{r=R}. \tag{20}$$

(19) va (20) dan quydagiga kelamiz

$$q_\ell\big|_{L^-} = -\varepsilon_s^0\, 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{r=R_L^-} - \left(1-\varepsilon_s^0\right) 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{x=R}. \tag{21}$$

$x = R_L^+$ sohada esa suspensiyadagi qattiq zarralar va suyuqlik tezliklari bir xil bo’ladi, uya’ni

$$\frac{q_s\big|_{L^+}}{\varepsilon_{s_0}} = \frac{q_\ell\big|_{L^+}}{1-\varepsilon_{s_0}}.$$

Natijada

$$q_s\big|_{L^-} = \frac{\varepsilon_{s_0}}{1-\varepsilon_{s_0}} q_\ell\big|_{L^+}. \tag{22}$$

(3) va (22) munosabatlarda

$$q_\ell\big|_{L^+} = -\left(1-\varepsilon_{s_0}\right) 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{r=R} \tag{23}$$

kelamiz.

(21), (23) munosabatlarni (16) qo‘yib quyidagi tenglamaga kelamiz

$$2\pi R_L \frac{dR_L}{dt} = \frac{\varepsilon_s^0}{\varepsilon_s^0 - \varepsilon_{s_0}}\left[2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{r=R_L^-} - 2\pi r \frac{k}{\mu}\frac{\partial p_\ell}{\partial r}\bigg|_{r=R}\right]. \tag{24}$$

p_ℓ va p_s orasidani (13) munosabatni qullasak, (24) tenglamani p_s ga nisbatan qaytadan yozish mumkin

$$\frac{dR_L}{dt} = -\frac{\varepsilon_s^0}{\varepsilon_s^0 - \varepsilon_{s_0}} \frac{k}{\mu}\frac{\partial p_s}{\partial r}\bigg|_{r=R_L^-} - \frac{1}{2\pi R_{L^-}} q_{\ell m}, \tag{25}$$

bu yerda

$$q_{\ell m} = -\left. 2\pi r\frac{k}{\mu}\frac{\partial p_s}{\partial r}\right|_{r=R}$$

(25) tenglama harakatlanuvchi chegarami aniqlaydi. $R_L(t)$ - suspenziya va cho‘kma qatlami orasidagi chegara.

Jarayon boshlanishidan oldin filtr toza bo’ladi, ya’ni

$$R_L(0) = R. \qquad (26)$$

Jarayon uchun boshlang‘ich shart quydagicha bo’ladi

$$p_\ell(0,r) = 0,\; p_s(0,r) = 0. \qquad (27)$$

Chegara shartlari esa quyidagicha olish mumkin:
Bir xil bosim berilganda

$$r = R_L(t) \text{ da } p_\ell = p_0,\; p_s = 0,\; \varepsilon_s = \varepsilon_s^0, \qquad (28)$$

$$r = R \text{ da } -2\pi r\frac{k}{\mu}\frac{\partial p_\ell}{\partial r} = \frac{-p_\ell}{R_m\mu}. \qquad (29)$$

Bir xil suyuqlik sarfi bo‘lganda

$$r = R_L(t) \text{ da } p_s = 0,\; \varepsilon_s = \varepsilon_s^0, \qquad (30)$$

$$r = R \text{ da } -2\pi r\frac{k}{\mu}\frac{\partial p_\ell}{\partial r} = -\frac{-p_\ell}{R_m\mu} = const. \qquad (31)$$

O’zgaruvchan bosim bo‘lganda;

$$r = R_L(t) \text{ da } p_\ell = p(t),\; p_s = 0,\; \varepsilon_s = \varepsilon_s^0, \qquad (32)$$

$$r = R \text{ da } -2\pi r\frac{k}{\mu}\frac{\partial p_\ell}{\partial r} = -\frac{p_\ell}{R_m\mu}. \qquad (33)$$

Silindr koordinatalarida cho‘kma qatlamini hosil bo‘ladigan suspenziyalarni filtrlashning asosiy tenglamasi, suspenziya va cho‘kma qatlami orasidagi chegarani toppish tenglamalari boshlang‘ich va chegaraviy shartlar bilan birgalikda yechiladi. Bu tenglamalar sistemasini yechish uchun sonli usullardan foydalanmiz. Buning uchun tenglamalar sistemasini yechish uchun quyidagi soddalashtirishlarni amalga oshiramiz.

Bir xil bosim rejimini qaraylik. U holda (28) va (29) chegaraviy shartlarini hisobga olamiz va (28), (29) larni p_s ga nisbatan quyidagicha yozib olamiz:

$$-2\pi \left. r\frac{k}{\mu}\frac{\partial p_s}{\partial r}\right|_{r=R} = \left.\frac{p_0 - p_s}{\mu R_m}\right|_{r=R},\quad p_s(t, R_L(t)) = 0. \qquad (34)$$

(14) tenglamani qayta yozib

$$\frac{\partial p_s}{\partial t} = \frac{p_A k^0}{\beta\mu}\left(1+\frac{p_s}{p_A}\right)^{1-\beta}\frac{1}{r}\frac{\partial}{\partial r}\left(\left(1+\frac{p_s}{p_A}\right)^{\beta-\delta} r\frac{\partial p_s}{\partial r}\right) + \frac{q_{out}}{2\pi}\frac{1}{r}\frac{\partial p_s}{\partial r} \qquad (37)$$

va quyidagi belgilashlani kiritsak,

$$a(p) = \frac{p_A k^0}{\beta\mu}\left(1+\frac{p_s}{p_A}\right)^{1-\beta},\; b(p) = \left(1+\frac{p_s}{p_A}\right)^{\beta-\delta},\; c(p) = \frac{\varepsilon_s^0}{\varepsilon_s^0 - \varepsilon_{s_0}}\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)^{-\delta}.$$

$$c^0(p) = \left.\frac{k^0}{\mu}\left(1+\frac{p_s}{p_A}\right)^{-\delta}\right|_{r=R}$$

(37) quyidagi ko‘rinishga keladi

$$\frac{\partial p_s}{\partial t} = a(p)\frac{1}{r}\frac{\partial}{\partial r}\ b(p)r\frac{k}{\mu}\frac{\partial p_s}{\partial r}\ -\frac{q_{out}}{2\pi}\ \frac{1}{r}\frac{\partial p_s}{\partial r}\ . \tag{38}$$

$R_L(t)$ - suspenziya va cho‘kma qatlami orasidagi chegaraning o‘zgarishi belgilashlarda quyidagi shaklga keladi

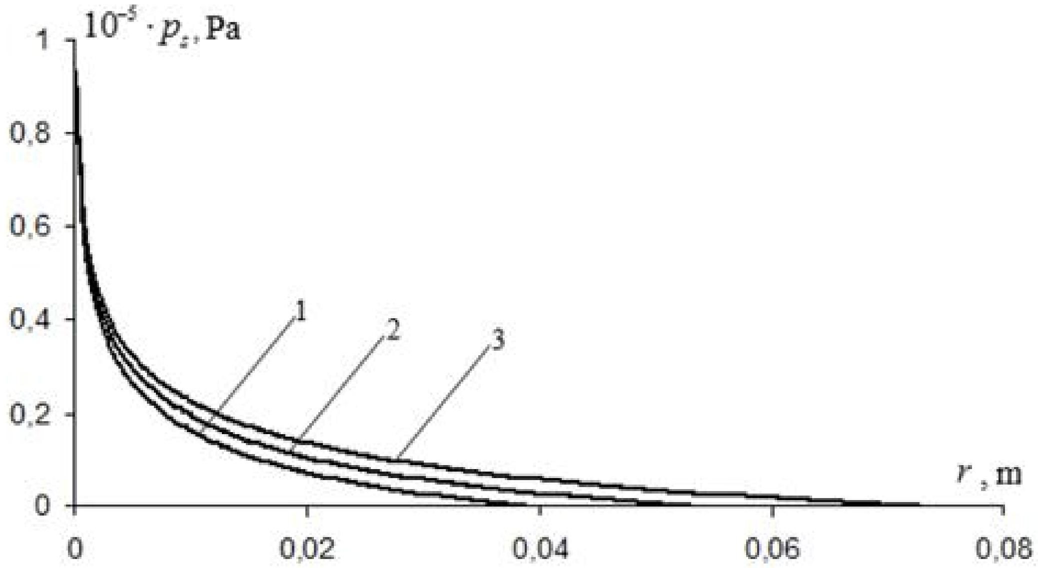
$$\frac{dR_L}{dt} = c(p)\ \frac{\partial p_\ell}{\partial r}\bigg|_{r=R_{L^-}} + \frac{1}{2\pi R_{L^-}} q_{out}\ , \tag{39}$$

bu yerda

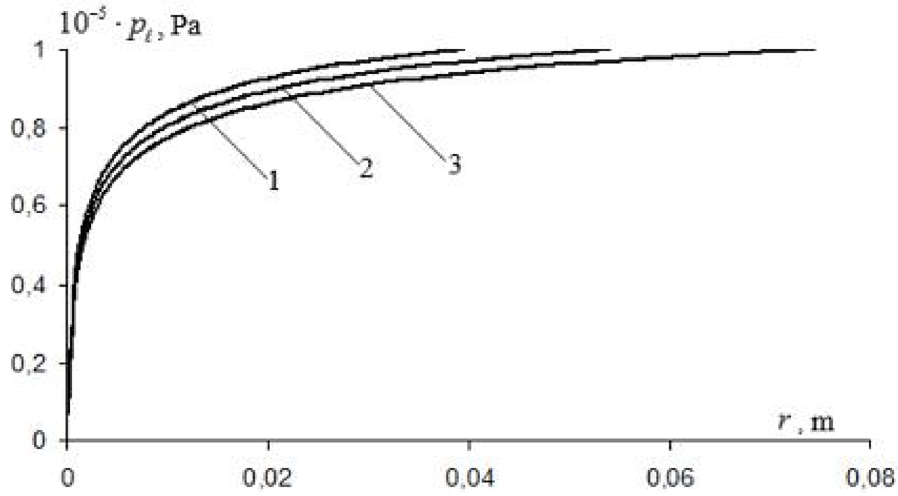
$$q_{out} = -c^0(p)2\pi r\frac{\partial p_\ell}{\partial r}\bigg|_{r=R}.$$

Quyilgan tenglamalar sistemasini yechish uchun chekli ayirmalar usulidan foydalanamiz [8,9]. Tenglamalar sistemasi quyidagi parametrlarning qiymati hisobga sonli yechildi: $p_A = 10^4$ Pa $p_0 = 10^5$ Pas $R_m = 10^{12}$ 1/ m², $\mu = 10^{-3}$ Pa s , $k^0 = 10^{-13}$ m2, $\varepsilon_s^0 = 0.20$, $\varepsilon_{s_0} = 0,0076$, $\beta = 0,13$, $\delta = 0,57$. Sonli natijalar 1 - 2 rasmda keltirilgan. Oligan natijalardan ko’rinib turibdiki, filtrlash jarayoni davomda cho‘kma qatlamining qalinligi oshadi hamda cho‘kma qatlamining qalinligi bo’ylab qattiq zarralar bosimi va suyuqlik bosimining taqsimlanishi belgilanadi. Qattiq zarralar bosimi filtrning yuzasidan cho‘kma qatlami va suspenziya chegarasiga qarab kamayadi. 1- va 2-rasmlarda harakatlanuvchi qatlam bilan aniqlanadigan aniq tugashga ega $R_L(t)$.

Shunday qilib, taqdim etilgan model suspenziyalarni silindr filtrlar orqali filtrlashning asosiy ko’rsatkichlarini to’g’ri hisoblash va shuningdek, cho‘kma qatlamini shakllantirish imkonini beradi degan xulosaga kelish mumkin.



1-rasm. Vaqtning turli qiymatlarida cho’kindi qalinligi bo’yicha suspenziyadagi qattiq zarralar bosimining taqsimoti. $t = 450$ (1); 900 (2); 1800 (3) s. ($r = R$ dan hisoblangan).



2-rasm. Vaqtning turli qiymatlarida cho'kindi qalinligi bo'yicha p_ℓ suyuqlik fazasi bosimining taqsimoti. $t = 450$ (1); 900 (2); 1800 (3) s. ($r = R$ dan hisoblangan).

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