SKALYAR ARGUMENTLI VEKTOR FUNKSIYA VA UNING XOSSALARI

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ABSTRACT

agar t=t0 nuqtada x(t), y(t), z(t) funksiyalar limitga ega bo`lsa, (t) vektor funksiyaning t=t0 nuqtadagi limiti

KEY WORDS

vektor, funksiya, radius vektor, funksiani hosilasi, funsiyani limiti

$$\lim_{t\to t_0}\vec{r}(t)=\lim_{t\to t_0}x(t)\vec{i}+\lim_{t\to t_0}y(t)\vec{j}+\lim_{t\to t_0}z(t)\vec{k}$$

bo`ladi.

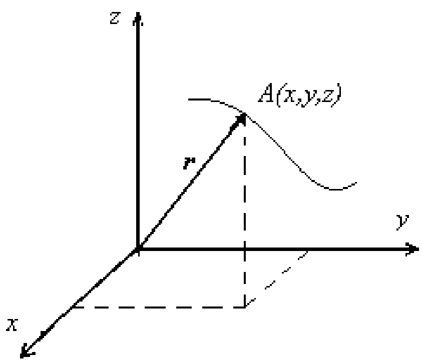
Skalyar argumentli vektor funksiya

***Ta`rif:*** Agar E sohadan olingan har bir haqiqiy $t$ songa biror qoidaga ko`ra bittadan $\vec{r}$ *(t)* vektor mos qo`yilgan bo`lsa, E to`plamda $t$ haqiqiy o`zgaruvchining vektor funksiyasi berilgan deyiladi.

Agar R^3 fazodagi bazis ($\vec{i}$, $\vec{j}$, $\vec{k}$) bo`lsa, u holda vektor funksiyani

$$\vec{r}(t)=x(t)\,\vec{i}+y(t)\,\vec{j}+z(t)\,\vec{k} \qquad (1)$$

ko`rinishda yozish mumkin. Bunda *x(t), y(t), z(t)* lar $\vec{r}$ vektorning koordinata o`qlaridagi proeksiyalaridir. Vektor funksiyaning berilishi bilan uchta skalyar funksiya *x(t), y(t), z(t)* larning berilishi teng kuchlidir.Agar $\vec{r}$ *(t)* vektoring boshlang`ich nuqtasi koordinatalar boshiga joylashtirilsa (bunday vektor radius-vektor deb ataladi), u holda $\vec{r}$ *(t)* vektor uchlarining geometrik



1-rasm

o`rni vektor funksiyaning godografi deyiladi. Godografning fizik ma`nosi shundan iboratki, agar t parametr vaqt deb olinsa, $\vec{r}$ *(t)* radius-vektorning godografi harakatdagi nuqtaning traektoriyasini bildiradi. (1-rasm)

Vektor funksiyaning hosilasi.

Agar $t=t_0$ nuqtada *x(t), y(t), z(t)* funksiyalar limitga ega bo`lsa, $\vec{r}$ *(t)* vektor funksiyaning $t=t_0$ nuqtadagi limiti

$$\lim_{t\to t_0}\vec r(t)=\lim_{t\to t_0}x(t)\vec i+\lim_{t\to t_0}y(t)\vec j+\lim_{t\to t_0}z(t)\vec k \qquad (2)$$

bo`ladi.

Agar $\lim_{t\to t_0}\vec r(t)=\vec r(t_0)$ bo`lsa, vektor-funksiya $t=t_0$ da uzluksiz deyiladi.

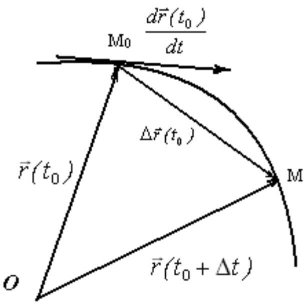
Endi $\vec r$ *(t)* vektor-funksiyaning hosilasi haqidagi masalaga o`tamiz.

Δ $\vec r(t_0)$ vektorning boshi koordinatalar boshida deb faraz qilamiz. Bu holda $\vec r$ *(t)* vektor-funksiyaning godografi parametrik ko`rinishda $x=x(t)$, $y=y(t)$, $z=z(t)$ tengliklar bilan berilgan fazoviy egri chiziqdan iborat bo`ladi. O`zgaruvchi t ning shu egri chiziqdagi M_0 nuqtaga mos keladigan $t=t_0$ qiymatini olib, unga Δt orttirma beramiz. U vaqtda

$$\vec r(t_0+\Delta t) \quad = x(t_0+\Delta t)\vec i+y(t_0+\Delta t)\vec j+z(t_0+\Delta t)\vec k$$

vektorni hosil qilamiz, bu vektor egri chiziqda biror M nuqtani aniqlaydi.(2-rasm).

Vektor-funksiya orttirmasini tuzamiz va uning skalyar argument orttirmasiga nisbatini qaraymiz:



2 - rasm

$$\frac{\Delta\vec r}{\Delta t}=\frac{\vec r(t_0+\Delta t)-\vec r(t_0)}{\Delta t}=\frac{x((t_0+\Delta t)-x(t_0)}{\Delta t}\vec i+\frac{y(t_0+\Delta t)-y(t_0)}{\Delta t}\vec j+\frac{z(t_0+\Delta t)-z(t_0)}{\Delta t}\vec k \qquad (3)$$

*Ta`rif*. Agar $\Delta t\to 0$ da $\frac{\Delta\vec r}{\Delta t}$ nisbatning chekli limiti mavjud bo`lsa, u limit $\vec r$ *(t)* vektor-funksiyaning $t=t_0$ nuqtadagi hosilasi deyiladi va $\vec r$ *`(t₀)*$\,$ yoki $\frac{d\vec r(t_0)}{dt}$ orqali belgilanadi.

$$\vec r\,'(t_0)=\lim_{\Delta t\to 0}\frac{\Delta\vec r}{\Delta t} \qquad (4)$$

Hosila vektorning yo`nalishini aniqlash maqsadida chizmaga e`tibor bersak, $t\to t_0$ da M nuqta M_0 ga, M_0M kesuvchi esa urinmaga intiladi. Demak, hosila vektor $\vec r\,'(t_0)$ parametrning o`sish tomoniga urinma bo`ylab yo`nalgan vektor bo`ladi.

Ravshanki, (10.3) tenglikdan $\vec r\,`(t_0)=$ $x'(t_0)\vec i+y'(t_0)\vec j+z'(t_0)\vec k$ ekanligi, bundan esa hosilani hisoblashning asosiy qoidalari vektor-funksiyalar uchun ham o`z kuchida qolishi kelib chiqadi.

Masalan: vektor-funksiyalar yig`indisining hosilasi qo`shiluvchi vektor-funksiyalar hosilalarining yig`indisiga teng.

Xususan, ikki vektor-funksiyalar yig`indisi uchun

$$(\vec r_1(t)+\vec r_2(t))'=\vec r_1\,'(t)+\vec r_2\,'(t)) \qquad (5)$$

ko`rinishdagi formula o`rinlidir.

Shunga o`xshash, O`zgarmas son ko`paytuvchisini hosila ishorasidan tashqariga chiqarish mumkin:

$$\frac{d(a\vec{r}(t))}{dt} = a\frac{d\vec{r}}{dt} \qquad (6)$$

Endi vektor-funksiyalarga xos amallar bilan bog`liq bo`lgan hosilani hisoblashning ba`zi qoidalarini keltiramiz. Bu qoidalarning isbotini o`quvchilarga mashq sifatida qoldiramiz.

1. Vektor-funksiyalarning skalyar ko`paytmasidan olingan hosila ushbu formula bilan ifodalanadi:

$$\frac{d(\vec{r}_1 \cdot \vec{r}_2)}{dt} = \frac{d\vec{r}_1}{dt}\vec{r}_2 + \vec{r}_1\frac{d\vec{r}_2}{dt} \qquad (7)$$

2. Agar *f(t)* skalyar funksiya va $\vec{r}$ *(t)* vektor-funksiya bo`lsa, *f(t)* $\vec{r}$ · *(t)* ko`paytmaning hosilasi ushbu formula bo`yicha hisoblanadi:

$$\frac{d(f(t)\vec{r}(t))}{dt} = \frac{df}{dt}\vec{r} + f\frac{d\vec{r}}{dt} \qquad (8)$$

3. $\vec{r}$ *₁(t)* va $\vec{r}$ *₂(t)* vektor-funksiyalarning vektor ko`paytmasining hosilasi

formula bo`yicha topiladi. $$\frac{d(\vec{r}_1 \times \vec{r}_2)}{dt} = a\frac{d\vec{r}_1}{dt} \times \vec{r}_2 + \vec{r}_1 \times \frac{d\vec{r}_2}{dt} \qquad (9)$$

Vektor funksiya

Birorta G to`plam berilgan bo`lsin ,Agar G to`plamning har bir nuqtasiga aniq bitta vector mos qo`yilgan bo`lsa ,G to`plamda vector funksiya berilgan deyiladi.Bu moslikni $p \Rightarrow \vec{r}(p)$ ko`rinishda yozing.

Ta`rif-1.

Berilgan $\vec{r}(p)$ vector funksiya va o`zgarmas $\vec{a}$ vector uchun $p \to p_0$ da $|\vec{r}(p) - \vec{a}| \to 0$ Munosabat bajarilsa, $\vec{r}(p)$ vector $p \to p_0$ da $\vec{a}$ limitga ega deyiladi va $\vec{r}(p) \to \vec{a}$ ko`rinishda yoziladi.

Bu yerda $|\vec{a}| = \sqrt{(\vec{a}, \vec{a})}$ bo`lib, $(\vec{a}, \vec{a})$ esa skalyar ko`paytmadir.

Agar formulada kiritilgan dekart kordinatalar sistemasida

$$\vec{r}(p) = \{x(p), y(p), z(p)\}, \vec{a} = \{a_1, a_2, a_3\} \text{ bo`lsa}$$

$p \to p_0$ da $\vec{r}(p) \to \vec{a}$ munosabat quyidagi uchta munosabatga ekvivalentdir.

$$\begin{array}{ccc} x(p) \to a_1 & y(p) \to a_2 & z(p) \to a_3 \\ p \to p_0 & p \to p_0 & p \to p_0 \end{array}$$

Vector funksiya limiti uchun quyidagi teoremani o`rinlidir.

*Ta`rif-6*. Berilgan $\vec{r}(p), \vec{p}(p)$ vector funksiyalar va $\lim\limits_{p \to p_0} \lambda(p) = \lambda_0$ tenglik bajarilsa, quyidagi munosabat o`rinlidir.

$$1)\ r(p) \pm \vec{p}(p) \xrightarrow[p \to p_0]{} \vec{a} \pm \vec{b}$$

$$2)\ \lambda(p) r(p) \xrightarrow[p \to p_0]{} \vec{a}$$

$$3)\ (r(p, p(p)) \xrightarrow[p \to p_0]{} (a, b)$$

$$4)\ [r(p), p(p) \xrightarrow[p \to p_0]{} [a, b]$$

1) $r(p) \pm p(p)$

$$\lim_{p\to p_0}(r(p)\pm p(p)) = a \pm b \Rightarrow \lim_{p\to p_0} r(p) = a, \ \lim_{p\to p_0} p(p) = b$$

isboti.

$$d(p) = r(p) \pm p(p), \ \vec{c} = \vec{a} + \vec{b}$$

$$\lim_{p\to p_0} |d(p) - c| = 0$$

$$|d(p) - c| = |(r(p) - a) \pm (p(p) - b)| \le |r(p) - a| \pm |p(p) - b|$$

$$0 \le 0$$

2) $\lim_{p\to p_0} \lambda(p) r(p) = \lambda_0 a$

$$|\lambda(p)r(p) - \lambda_0 a| \xrightarrow[p\to p_0]{} 0$$

$$|\lambda(p)r(p) + \lambda(p)a - \lambda(p)a| = |(\lambda(p)r(p) - a) + a(\lambda(p) - \lambda_0)| \le$$

$$|(\lambda(p)r(p) - a)| + |a(\lambda(p) - \lambda_0)| = 0$$

3) $(r(p)p(p)) \xrightarrow[p\to p_0]{} (a,b)$

$$r(p) = \{x_1(p), y_1(p), z_1(p)\} \quad \underset{p\to p_0}{x_1(p) \Rightarrow a_1}, \quad \underset{p\to p_0}{y_1(p) \Rightarrow a_2}, \quad \underset{p\to p_0}{z_1(p) \Rightarrow a_3},$$

$$p(p) = \{x_2(p), y_2(p), z_2(p)\} \quad \underset{p\to p_0}{x_2(p) \Rightarrow b_1}, \quad \underset{p\to p_0}{y_2(p) \Rightarrow b_2}, \quad \underset{p\to p_0}{z_2(p) \Rightarrow b_3},$$

$$a = \{a_1, a_2, a_3\}$$
$$b = \{b_1, b_2, b_3\}$$

$$r(p)p(p) = \{x_1(p)x_2(p) + y_1(p)y_2(p) + z_1(p)z_2(p)\}$$
$$r(p)p(p) = (x_1(p), y_1(p), z_1(p)) \cdot (x_2(p), y_2(p), z_2(p)) = x_1(p)x_2(p) + y_1(p)y_2(p) + z_1(p)z_2(p)$$

$$= a_1b_1 + a_2b_2 + a_3b_3 = (a,b)$$

1) $(\lambda(p)r(p))' = \lambda'(p)r(p) + \lambda(p)r'(p)$
2) $(r(p) \pm p(p))' = r'(p) \pm p'(p)$
3) $(r(p), p(p))' = (r'(p), p(p)) + (r(p), p'(p))$
4) $[r(p), p(p)]' = [r'(p), p(p)] + [r(p), p'(p)]$

Isboti.

1)
$$\lim_{h\to 0}\lambda(p)r(p)=\frac{\lambda(p+h)r(p+h)-\lambda(p)r(p)}{h}=$$
$$\frac{(\lambda(p+h)-\lambda(p))r(p+h)+\lambda(p)r(p+h)-\lambda(p)r(p)}{h}=$$
$$\lambda'(p)r(p)+r'(p)\lambda(p)$$

2)
$$\lim_{h\to 0}f(r(p)\pm p(p))=\frac{f(r(p)\pm p(p)+h)-F(r(p)\pm p(p))}{h}=$$
$$\frac{(r(p+h)-r(p))+r(p)\pm(p(p+h)-p(p))+p(p)-(r(p)+p(p))}{h}=$$
$$r'(p)\pm p'(p)$$

3)
$$\lim_{h\to 0}(r(p),p(p))=\frac{r(p+h),p(p+h)-r(p)p(p)}{h}=$$
$$\frac{(r(p+h)-r(p))p(p+h)+r(p)p(p+h)-r(p)p(p)}{h}=$$
$$=r'(p)p(p)+p'(p)r(p)$$

4)
$$\lim_{h\to 0}(r(p),p(p))=\frac{r(p+h),p(p+h)-r(p)p(p)}{h}=$$
$$\frac{(r(p+h)-r(p))p(p+h)+r(p)p(p+h)-r(p)p(p)}{h}=$$
$$=r'(p)p(p)+p'(p)r(p)$$

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