CHIZIQLI OPERATORLAR YORDAMIDA n-DARAJALI MATRITSALARNI HISOBLASH

Usmonova Shaxlo
Baxramova Mahliyo
Sharof Rashidov nomidagi Samarqand Davlat Universiteti matematika fakulteti 2-kurs talabasi.
Sharof Rashidov nomidagi Samarqand Davlat Universiteti matematika fakulteti 1-kurs talabasi.
e-mail: usmonovamath@mail.ru
https://doi.org/10.5281/zenodo.14892015

ARTICLE INFO

Qabul qilindi: 10- Fevral 2025 yil
Ma’qullandi: 15- Fevral 2025 yil
Nashr qilindi: 19- Fevral 2025 yil

KEY WORDS

n- darajali matritsa, matritsa bosh diagonali, matritsaning xos son va xos vektorlari, teskari matritsa.

ABSTRACT

Ushbu maqoladada n- darajali matritsalarni hisoblashda chiziqli operatorlardan foydalanish usullari keltirilgan.

Algebraning asosiy tushunchalaridan biri bo‘lgan va amaliyotda keng tadbiqlarga ega bo‘lgan matritsalar nazariyasini o‘rganish har doim muhim va ahamiyatli bo‘lgan. Xususan, matritsaning darajalarini hisoblash ushbu nazariyaning dolzarb va aksar hollarda qiyinchilik yaratadigan bo‘limi hisoblanadi. Agar berilgan matritsa diagonal ko‘rinishda bo‘lsa, matritsaning darajalarini hisoblash oson va buni chiziqli operatorning xos qiymat va xos vektorlaridan foydalanib hisoblash mumkin bo‘ladi. Quyida ushbu usul uchun zarur bo‘lgan ta’rif va tushunchalar misollar yordamida tushuntirilgan.

Faraz qilaylik, B diagonal matritsa berilgan bo‘lsin:

$$B = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{pmatrix}.$$

U holda $\forall m \in \mathbb{N}$ uchun quyidagi tenglik o‘rinli bo‘ladi:

$$B^m = \begin{pmatrix} \lambda_1^m & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n^m \end{pmatrix}.$$

Agar berilgan A matritsa biror diagonal matritsaga o‘xshash bo‘lsa, ya’ni $\exists C$ $(detC \neq 0)$ va T matritsa topilib, $T = C^{-1}AC$ ($C -$ xos vektorlardan tuzilgan matritsa) tenglik bajarilsa, u holda quyidagi tenglikni hosil qilamiz:

$$A = CTC^{-1}. \qquad (1)$$

Bundan foydalanib quyidagini hisoblaymiz:

$$A^n = A \cdot A \cdot \ldots \cdot A.$$

Endi har bitta A matritsani o'rniga (1) tenglikni qo'yib hisoblaymiz:

$$A^n = CTC^{-1} \cdot CTC^{-1} \cdot \ldots \cdot CTC^{-1}.$$

Bundan C^1C ko'paytma birlik matritsani beradi.

Demak, bundan quyidagi natija kelib chiqadi:

$$A^n = CT^nC^{-1} = C\begin{pmatrix} \lambda_1^n & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n^n \end{pmatrix}C^{-1}.$$

1-misol. Quyidagi matritsani hisoblang:

$$A = \begin{pmatrix} 1 & 2 \\ -3 & 6 \end{pmatrix}^{2024}.$$

Yechimi. Dastlab matritsaning xos son va xos vektorlarini topamiz:

$$|A - \lambda E| = \begin{vmatrix} 1-\lambda & 2 \\ -3 & 6-\lambda \end{vmatrix} = 0.$$

Bundan $\lambda_1 = 4$ va $\lambda_2 = 3$ qiymatlarni topamiz va unga mos xos vektorlarni aniqlab, ulardan C matritsani tuzib olamiz:

$$C = \begin{pmatrix} 2 & 1 \\ 3 & 1 \end{pmatrix}, \quad C^{-1} = \begin{pmatrix} -1 & 1 \\ 3 & -2 \end{pmatrix}.$$

$$T = \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix}, \quad T^{2024} = \begin{pmatrix} 4^{2024} & 0 \\ 0 & 3^{2024} \end{pmatrix}.$$

$$A^{2024} = C\begin{pmatrix} 4^{2024} & 0 \\ 0 & 3^{2024} \end{pmatrix}C^{-1}.$$

2-misol. $\mathbb{R}^4$ fazoda A operatorning matritsasi berilgan bo'lsa,

$$A = \begin{pmatrix} 3 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 3 & 0 & -5 & -3 \\ 4 & -1 & 3 & 1 \end{pmatrix}$$

shu matritsaning 10-darajasi hisoblansin.

Yechimi. Matritsaning xos qiymat va xos vektorlarini topib olamiz:

$$\begin{vmatrix} 3-\lambda & -1 & 0 & 0 \\ 1 & 1-\lambda & 0 & 0 \\ 3 & 0 & -5-\lambda & -3 \\ 4 & -1 & 3 & 1-\lambda \end{vmatrix} = 0.$$

Quyidagi determinantni hisoblab, unga mos λ larni topib olamiz:

$$\begin{vmatrix} \lambda^2 - 4\lambda + 4 & 0 & 0 \\ 3 & -5-\lambda & -3 \\ 1+\lambda & 3 & 1-\lambda \end{vmatrix} = (\lambda-2)^2(\lambda+2)^2 = 0.$$

Demak,

$$\lambda_1 = 2, \;\; \lambda_2 = 2, \;\; \lambda_3 = -2, \;\; \lambda_4 = -2.$$

$\lambda = 2$ bo’lganda

$$A - 2E = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 3 & 0 & -7 & -3 \\ 4 & -1 & 3 & -1 \end{pmatrix}.$$

U holda xos vektorlarning koordinatalari ushbu

$$\begin{cases} x_1 - x_2 = 0 \\ x_1 - x_2 = 0 \\ 3x_1 - 7x_3 - 3x_4 = 0 \\ 4x_1 - x_2 + 3x_3 - x_4 = 0 \end{cases}.$$

Bu sistemaning yechimlari bitta vektordan iborat:

$$a_{1,2} = (8, 8, -3, 15).$$

Endi $\lambda = -2$ bo’lganda xos vektorlarni aniqlaymiz:

$$A + 2E = \begin{pmatrix} 5 & -1 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 3 & 0 & -3 & -3 \\ 4 & -1 & 3 & 3 \end{pmatrix} \to \begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}.$$

Bundan xos vektorlar koordinatalari

$$\begin{cases} x_1 + 3x_2 = 0 \\ x_2 = 0 \\ x_3 + x_4 = 0 \end{cases}$$

tenglamalar sistemasidan aniqlanadi. Oxirgi sistemaning fundamental yechimlari bitta vektordan iborat.

$$a_{3,4} = (0, 0, -1, 1).$$

Xos vektorlardan tuzilgan matritsa quyidagicha

$$C = \begin{pmatrix} 8 & 8 & 0 & 0 \\ 8 & 8 & 0 & 0 \\ -3 & -3 & -1 & -1 \\ 15 & 15 & 1 & 1 \end{pmatrix}.$$

C matritsa determinanti noldan farqli demak teskarisi mavjud.

Bundan

$$A^{10} = C \begin{pmatrix} 2^{10} & 0 & 0 & 0 \\ 0 & 2^{10} & 0 & 0 \\ 0 & 0 & (-2)^{10} & 0 \\ 0 & 0 & 0 & (-2)^{10} \end{pmatrix} C^{-1}.$$

3-misol. Quyidagini matritsani hisoblang:

$$A = \begin{pmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{pmatrix}^n.$$

Bu matritsani ham xos qiymat va unga mos bo'lgan xos vektorlarini topamiz:

$$\begin{vmatrix} -1-\lambda & 3 & -1 \\ -3 & 5-\lambda & -1 \\ -3 & 3 & 1-\lambda \end{vmatrix} = 0.$$

Determinantni hisoblab soddalashtiramiz va λ larni aniqlaymiz:

$$\lambda^3 - 5\lambda^2 + 8\lambda - 4 = 0,$$

$$\lambda_1 = 1 \quad va \quad \lambda_{2,3} = 2.$$

Bu yerda λ_2 ikki karrali ildiz hisoblanadi. Demak, endi mos C matritsani topamiz:

$$C = \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix},$$

$$C^{-1} = \begin{pmatrix} -2 & -3 & 3 \\ 3 & 4 & -3 \\ -1 & -1 & -1 \end{pmatrix},$$

$$A^n = C \begin{pmatrix} 1^n & 0 & 0 \\ 0 & 2^n & 0 \\ 0 & 0 & 2^n \end{pmatrix} C^{-1}.$$

4-misol. Quyidagi matritsaning 1000-darajasi topilsin:

$$A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 0 \end{pmatrix}.$$

Yechim. Matritsaning xos qiymat xos vektorlarini topamiz:

$$|A - \lambda E| = 0,$$

$$\begin{vmatrix} 1-\lambda & -1 & 1 \\ 1 & 1-\lambda & -1 \\ 2 & -1 & -\lambda \end{vmatrix} = 0.$$

Determinantni hisoblasak, $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = -1$ qiymatlar kelib chiqadi. Endi λ xos qiymatlarga mos B diagonal matritsani tuzib olamiz:

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Xos vektorlarini topamiz:

$\lambda_1 = 2$ ni qo'yib hisoblaganimizda unga mos bo'lgan xos vektorlari $y_1 = (1, 0, 1)$

$\lambda_2 = 1$ ni qo'ysak, $y_2 = (1, 1, 1)$

$\lambda_3 = -1$ dagi qiymati $y_3 = (-1, 3, 5)$

Xos vektorlarni qo'yib, C matritsani tuzamiz:

$$C = \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 3 \\ 1 & 1 & 5 \end{pmatrix},$$

$$C^{-1} = \begin{pmatrix} \frac{1}{3} & -1 & \frac{2}{3} \\ \frac{1}{2} & \frac{5}{6} & -\frac{1}{2} \\ -\frac{1}{6} & 0 & \frac{1}{6} \end{pmatrix},$$

$$A^{1000} = CB^{1000}C^{-1},$$

$$A^{1000} = C\begin{pmatrix} 2^{1000} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}C^{-1}.$$

5-misol. Quyidagi matritsaning 8-darajasi hisoblansin:

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$

Yechimi. Bu matritsaning xarakteristik tenglamasidan λ larni topamiz:

$$\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = 3.$$

Bu xos qiymatlarga mos keladigan xos vektorlarni topamiz:

$$u_1 = (-1, 0, 1), u_2 = (0, 1, 0), u_3 = (1, 1, 1).$$

Xos vektorlarni transponerlab, C matritsani tuzamiz:

$$C = \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

Bu matritsaning determinanti noldan farqliligidan uning teskari mavjud bo’ladi. Demak,

$$A^8 = C\begin{pmatrix} 1^8 & 0 & 0 \\ 0 & 2^8 & 0 \\ 0 & 0 & 3^8 \end{pmatrix}C^{-1}.$$

Foydalanilgan adabiyotlar ro’yxati:

1.Ж.Хожиев, А.С.Файнлейб Алгебра ва сонлар назария курси. ТОШКЕНТ:УЗБЕКИСТОН,2001.
2.D.I.Yunusova, A.S.Yunusov Algebra va sonlar Nazariyasi misol va masalalar to’plami. ILM-ZIYO:TOSHKENT,2009.
3.Sh.A.Ayupov, B.A.Omirov, A.X.Xudoyberdiyev, F.H.Haydarov Algebra va sonlar nazariyasi. Toshkent,2019.
4.А.Г.Курош Олий алгебра курси. Тошкент: Укитувчи,1976.
5.Р.Искандаров Олий алгебра, Тошкент: Укитувчи, 1960.