

THE GENERAL RELATIVITY IN HIGHER DIMENSION IS DESCRIBED BY THE GENERALIZED EINSTEIN-HILBERT ACTION

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91-202-06-76

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<https://doi.org/10.5281/zenodo.8115569>

Introduction

A black hole is one of the most fascinating object whose existence was predicted by general theory of relativity a century ago, as it can be described by the only three parameters: mass, angular momentum and charge, according to the nohair theorem. Its existence has recently been proven thanks to the technological development by the detected gravitational wave signals from coalescence of black holes/neutron stars in binary [1–6] by the LIGO-Virgo collaborations and by the observational evidence based on electromagnetic radiation coming from accretion disks around astrophysical black holes [7] as well as from the detected first ever image of the supermassive black hole at the center of the galaxy M87 by the Event we summarize main results obtained in the paper. Throughout the paper we use the geometrized units in which the Newtonian gravitational constant G , speed of light c are set as $G = c = 1$.

The Tasks of the research:

1. to study deflection angle of the light ray around black hole and its images, brightness of the compact objects using gravitational lensing effect;
2. to estimate the emission energy from the black hole through the thermal radiation;
3. to consider the collision of particles near the black hole;

The general relativity coupled to the scalar field generating global monopole is described by the action

$$S = \int d^4x \sqrt{-g} (R + L_{GM}), \quad (1)$$

where the Lagrangian density of global monopole (GM) is given by

$$\mathcal{L}_{GM} = \frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{\lambda}{4} (\phi^a \phi^a - \eta^2)^2, \quad (2)$$

where the scalar field is given by

$$\phi^a = \eta f(r) \frac{x^a}{r}, \quad a = 1, 2, 3, \quad (3)$$

where x^a is Cartesian coordinates ($x^a x^a = r^2$), λ is the selfinteraction term. As one noticed without loss of generality we adopted units such that the gravitational constant and speed of the light are equal to unit i.e. $G = c = 1$. After applying the least action principle to the action (1) we obtain Einstein field equations as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}, \quad (4)$$

where $R_{\mu\nu}$, R , and $g_{\mu\nu}$ are Ricci tensor, Ricci scalar and metric tensor, respectively. The energy-momentum tensor of the global monopole is given by

$$\begin{aligned} T_t^t &= -\eta^2 \left(\frac{f^2}{r^2} + \frac{N^2 f'^2}{2} \right) - \frac{\eta^4 \lambda}{4} (f^2 - 1)^2, \\ T_r^r &= -\eta^2 \left(\frac{f^2}{r^2} - \frac{N^2 f'^2}{2} \right) - \frac{\eta^4 \lambda}{4} (f^2 - 1)^2, \\ T_\theta^\theta &= -\frac{\eta^2 N^2 f'^2}{2} - \frac{\eta^4 \lambda}{4} (f^2 - 1)^2 = T_\varphi^\varphi. \end{aligned} \quad (5)$$

By solving the Einstein field equations (4) for the spacetime with topology $R \times S^2$ one can obtain the following Schwarzschild black hole solution with global monopole [12]:

$$ds^2 = -N^2(r) dt^2 + \frac{dr^2}{N^2(r)} + r^2 d\Omega^2, \quad (6)$$

where

$$N^2(r) = 1 - \eta^2 - \frac{2M}{r}, \quad (7)$$

and the line element of two dimensional sphere is given by

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2, \quad (8)$$

We must here note that this solution is asymptotically non-flat. The event horizon of the spacetime is given by the divergence of the spacetime metric through g_{rr} that gives

$$r_h = \frac{2M}{1 - \eta^2}. \quad (9)$$

From expression (9) one can see that when the parameter η vanishes, one recovers the event horizon of the Schwarzschild black hole, while when η tends to unity from the left side, the event horizon tends to infinity. For the values of the global monopole $\eta \geq 1$, the time and radial components of the metric tensor becomes space-like and time-like, respectively, that is beyond our considerations.

In this section we study a motion of the test particle around global monopole whose spacetime is described by the line element (6). For simplicity of our further calculations we assume the motion of the particle is confined at the equatorial plane, $\theta = \pi/2$, and by doing this we eliminate the equation of motion corresponding to the coordinate θ . To find the equations of motion of particle, we first find the conservative quantities from the symmetry of the spacetime metric (6) which are the energy, E , that is a momenta corresponding to the time, t , and the angular momentum, L , that is a momentum corresponding to the azimuthal coordinate, φ , given by

$$\begin{aligned} \left(1 - \eta^2 - \frac{2M}{r} \right) u^t &= E, \\ r^2 u^\varphi &= L. \end{aligned} \quad (10)$$

To find the remaining only component of the equation of motion we use the normalization condition $u_\mu u^\mu = -\epsilon$ and obtain the following equation for the radial velocity of the particle:

$$\begin{aligned} (u^r)^2 &= E^2 - V_{\text{eff}} \\ V_{\text{eff}} &= N^2(r) \left(\frac{L^2}{r^2} + \epsilon \right) \end{aligned} \quad (11)$$

where a notation ϵ represents a mass of the test particle that is either $\epsilon = 1$ for the massive particle or $\epsilon = 0$ for massless particle (photon), and V_{eff} represents the effective potential whose radial dependence is given in Fig. 1. From that radial profile of the effective potential one can see that with increasing the value of the global monopole, a height of the effective

potential decreases. However, on the other hand, the maximum of the effective potential which corresponds to the unstable circular photon orbit shifts further from the centre of the spacetime.

Now we focus on the circular orbits of the particle around global monopole. By using the well known fact that the particle moving along the circular orbit has no radial velocity and acceleration that are described by the conditions

$$u^r = 0, \quad V'_{\text{eff}} = 0, \quad (12)$$

one can find the specific energy and specific angular momentum of the test particle moving along the circular orbit around Schwarzschild black hole with global monopole as

$$\mathcal{E}^2 = -\frac{[(\eta^2 - 1)r + 2M]^2}{r[(\eta^2 - 1)r + 3M]}, \quad (13)$$

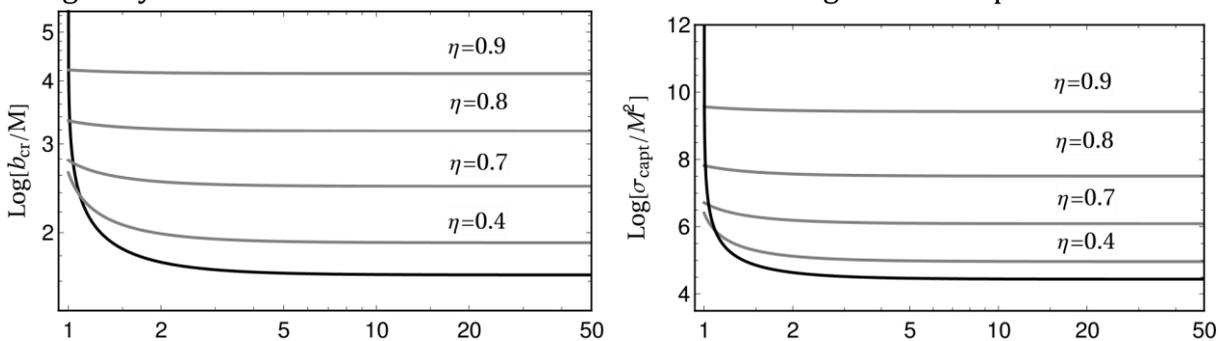
$$\mathcal{L}^2 = -\frac{Mr^2}{(\eta^2 - 1)r + 3M}. \quad (14)$$

Note that in eq. (13) we have written the expressions in terms of the specific energy (and specific angular momentum) of the particle that are defined by the energy (and angular momentum) corresponding to per mass of the particle as $E = E/\epsilon$ (or $L = L/\epsilon$). By taking the fact into account that the energy and angular momentum of the photon in the circular orbit become infinite, we find the radius of the photonsphere in the Schwarzschild black hole with global monopole as

$$r_{\text{ps}} = \frac{3M}{1 - \eta^2}, \quad (15)$$

Conclusion

In this paper we have studied the characteristic circular orbits of the massive and massless particles around Schwarzschild black hole with global monopole that is a solution of general relativity coupled to the Goldstone field generating global monopole. By studying the circular orbits of test particles, we have determined that the global monopole of the black hole strengthens the gravitational attraction of the black hole, consequently, its horizon and radii of all the characteristic circular orbits, such as radii of photonsphere, innermost stable and marginally bound circular orbits increase on account of the global monopole



Schwarzschild and other black holes of general relativity diverges. Interestingly, the capture cross section of such particle by the Schwarzschild black hole with global monopole does not diverge, unless $\eta = 1$, instead it tends to finite value. This finite value is given as a function of the global monopole in

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